Problem 1 (4 points) The language $\forall^* \exists^* \text{CALC}$ consists of all CALC sentences of the form

$$\forall y_1 \ldots \forall y_n \exists z_1 \ldots \exists z_m \varphi(y_1, \ldots, y_n, z_1 \ldots z_m)$$

where $\varphi$ is a quantifier-free CALC formula. Prove that satisfiability of sentences in $\forall^* \exists^* \text{CALC}$ is undecidable.

Problem 1 (4 points) Let $A_r$ be the directed graph consisting of one simple path of length $r$ (the length of a path is taken to be the number of edges along the path). Does Duplicator have a winning strategy for the Ehrenfeucht-Fraissé game of length 4 on $A_7$ and $A_8$?

Problem 2 (6 points) Determine whether the following properties of graphs are almost surely true or whether they are almost surely false.

(a) Existence of a cycle of length three.

(b) Connectivity.

(c) Being a tree.

Hint: Try to avoid brute-force counting; use what you already know.

Problem 4 (6 points) We say that a graph property is definable by a Datalog program $P$ with some designated IDB relation $\text{answer}$ and one binary EDB relation $G$ iff for every $G$, $\text{answer}$ is non-empty in $P(G)$ iff the property holds for $G$. Show the following:

(i) (3 points) The existence of a path of even length between two designated nodes $a$ and $b$ in a graph is definable in Datalog.

(ii) (3 points) The existence of a simple path of even length between two designated nodes $a$ and $b$ in a graph is not definable in Datalog.