1. (3 points) Prove that it is undecidable whether a CALC query is monotonic (recall that a query $Q$ is monotonic if for all databases $D_1, D_2$, if $D_1 \subseteq D_2$ then $Q(D_1) \subseteq Q(D_2)$).

2. (3 points) Let $Q_1, Q_2, Q_3$ be conjunctive queries (no equality). Prove that, if $Q_1 \subseteq (Q_2 \cup Q_3)$, then $Q_1 \subseteq Q_2$ or $Q_1 \subseteq Q_3$ (recall that the notation $P \subseteq R$ for queries $P, R$ means that $P(I) \subseteq R(I)$ for every database $I$).

3. (i) (4 points) The language $\exists^* \forall^* \text{CALC}$ consists of all CALC formulas of the form
$$\exists y_1 \ldots \exists y_n \forall z_1 \ldots \forall z_m \varphi(\bar{x}, y_1, \ldots, y_n, z_1 \ldots z_m)$$
where $\varphi$ is a quantifier-free CALC formula. Prove that satisfiability is decidable for queries in $\exists^* \forall^* \text{CALC}$. **Hint:** Come up with a bound on the size of databases one needs to look at in order to check satisfiability. More precisely, show that a query $Q$ in this language is satisfiable iff it is satisfied on some database of size bounded by $f(Q)$ for some computable function $f$.

(ii) (3 points) The language $CQ^-$ consists of queries of the form $\varphi(\bar{x}) - \psi(\bar{x})$ where $\varphi$ and $\psi$ are CQs. Prove that equivalence of $CQ^-$ queries is decidable.

4. (5 points) A database consists of the following binary relations:

\[
\begin{array}{c|cc}
P & A & B \\
\hline
Q & B & C \\
R & A & C \\
\end{array}
\]

($P$ with attributes $AB$, $Q$ with attributes $BC$ and $R$ with attributes $AC$). Consider the relational algebra query $P \bowtie Q \bowtie R$ (equivalent to the formula $\varphi(a, b, c) = P(a, b) \land Q(b, c) \land R(a, c)$). Assuming that $P, Q$ and $R$ have size at most $n$ (i.e. they each contain at most $n$ pairs), find the tightest upper bound you can on the size of $P \bowtie Q \bowtie R$, as a function of $n$.

**Hint:** There is an obvious upper bound of $n^2$. Try to do better.
5. (2 points) Recall the movie database in Problem 1 of the previous homework, and the query *List the theaters showing only movies by Hitchcock.* Express this query in nr-Datalog."