Problem 1 (6 points)

(a) Write the informal negation (in English) of the following statements:
1. “There is a CSE20 student who is not a sophomore”
2. “All cats are grey in the dark”
3. “Some pigs can fly”

(b) Express the following statements in symbolic form using the predicates $\text{CPU}(c) = \text{"c has a CPU"}$, $x^2 \geq 1$, $x > 0$, $E(x, y) = \text{"(x − y) is even"}$, and write their formal negation:
1. $\forall\text{ computers } c, \text{ has a CPU}$
2. $\forall\text{ real numbers } x, \text{ if } x^2 \geq 1 \text{ then } x > 0$
3. $\forall\text{ integers } a, b, c, \text{ if } (a − b) \text{ is even and } (b − c) \text{ is even, then } a − c \text{ is even.}$

Your expressions should be simplified so that no quantifier or connective lies within the scope of a negation.

Problem 2 (2 points)

For each of the following statements, determine whether the proposed negation is correct. If it is not, explain why and write a correct negation.
1. “The sum of any two irrational numbers is irrational”. Proposed negation: “The sum of any two irrational numbers is rational”
2. “For all integers $n$, if $n^2$ is even, then $n$ is even. Proposed negation: “For all integers $n$, if $n^2$ is even, then $n$ is odd”.

Problem 3 (6 points)

Let $D = \{-48, -14, -8, 0, 1, 3, 16, 23, 26, 32, 36\}$. Determine which of the following statements are true and which are false. Provide a counterexample for the statements that are false. In all statements, the variables $x, y$ range over the set $D$.
1. $\forall x, \text{ if } x \text{ is odd then } x > 0.$
2. $\forall x, \text{ if } x < 0 \text{ then } x \text{ is even}$
3. $\forall x, \text{ if } x \text{ is even, then } x \leq 0$
4. $\forall x, \exists y \text{ such that } y > x$
5. $\forall x, \text{ (x is even or } \exists y, y > x)$
6. $\forall x, \text{ (x is odd or } \exists y, y > x)$
Problem 4 (6 points)

Give a justification for each step in the following proof sequence showing that if \((q \land r)\) and \((\neg (\neg p \land q))\) are true, then \(p\) is true. Each justification should include the name of the inference rule used and which steps of the derivation it refers to.

1. \(q \land r\). Justification: Premise
2. \(\neg (\neg p \land q)\). Justification: ...
3. \(\neg \neg p \lor \neg q\)
4. \(p \lor \neg q\)
5. \(\neg q \lor p\)
6. \(q \rightarrow p\)
7. \(q\)
8. \(p\)

You can use any standard inference rule (or rule name) mentioned in the textbook or in class. For example, possible inference rules include “modus ponens”, “modus tollens”, “De Morgan”, “Double negation”, “Commutativity of X”, “Associativity of X”, where \(X\) is either \(\land\) or \(\lor\). If the rule uses previous statements in the sequence, make sure you indicate the steps you are applying the rule to.

Problem 5 (10 points)

In this problem, the domain is the set of all faces of a truncated icosahedron (also known as a soccer ball). If you don’t know what that looks like, just search the web for an image of a “soccer ball”. (Usually the hexagons are white and the pentagons are black.) Consider the following predicates:

- \(P(x) = \text{“}x\text{ is a pentagon”}\)
- \(H(x) = \text{“}x\text{ is an hexagon”}\)
- \(B(x,y) = \text{“}x\text{ borders }y\text{”, i.e., faces }x\text{ and }y\text{ share an edge.}\) (For the purpose of this exercise, a face is not considered to border itself, i.e., \(B(x,x)\) is always false.)

Consider the following statements:

- No two pentagons border each other
- Every pentagon borders some hexagon
- Every hexagon borders another hexagon
- Every two hexagons border each other
- Every two pentagon border each other
- There is a pentagon that borders only hexagons.

For each of the above statements, do the following:

(a) Indicate which of the above statements are true.

(b) Write the statements in predicate logic.

(c) Negate the statements from (b). Simplify the negated statements so that no quantifier or connective lies within the scope of a negation.

(d) Translate the negated statements back into English.