Image Formation:
Geometric Camera Models

Introduction to Computer Vision
CSE 152
Lecture 2
Announcements

• Course website
  – https://cseweb.ucsd.edu/classes/sp18/cse152-a/

• Homework 1 will be assigned today
  – Working with images in Python
  – Due Wed, Apr 11, 11:59 PM

• Wait list

• Reading:
  – Chapters 1: Geometric camera models
Earliest Surviving Photograph

- First photograph on record, “la table service” by Nicephore Niepce in 1822.
- Note: First photograph by Niepce was in 1816.
How Cameras Produce Images

• Basic process:
  – photons hit a detector
  – the detector becomes charged
  – the charge is read out as brightness

• Sensor types:
  – CCD (charge-coupled device)
    • high sensitivity
    • high power
    • cannot be individually addressed
    • blooming
  – CMOS
    • simple to fabricate (cheap)
    • lower sensitivity, lower power
    • can be individually addressed
Images are two-dimensional patterns of brightness values.

They are formed by the projection of 3D objects.
Effect of Lighting: Monet
Change of Viewpoint: Monet

Haystack at Chailly at sunrise (1865)
Image Formation: Outline

- Geometric camera models
- Light and shading
- Color
Pinhole Camera: Perspective projection

- Abstract camera model - box with a small hole in it
"When images of illuminated objects ... penetrate through a small hole into a very dark room ... you will see [on the opposite wall] these objects in their proper form and color, reduced in size ... in a reversed position, owing to the intersection of the rays". --- Leonardo *Da Vinci*

http://www.acmi.net.au/AIC/CAMERA_OBSCURA.html (Russell Naughton)
Camera Obscura

- Used to observe eclipses (e.g., Bacon, 1214-1294)
- By artists (e.g., Vermeer).
Camera Obscura

Jetty at Margate England, 1898.

http://brightbytes.com/cosite/collection2.html (Jack and Beverly Wilgus)
Geometry

- How do 3D world points project to 2D image points?
Distant objects are smaller

(Forsyth & Ponce)
Geometric properties of projection

- 3-D points map to **points**
- 3-D lines map to **lines**
- Planes map to **whole image** or half-plane
- Polygons map to **polygons**

- Important point to note: Angles & distances not preserved, nor are inequalities of angles & distances.

- Degenerate cases:
  - line through focal point project to **point**
  - plane through focal point projects to a **line**
In the perspective image, two parallel lines meet at a point.
Parallel lines meet in the image

- Formed by line through O
- Parallel to the given line(s)
- A single line can have a vanishing point
Projective geometry provides an elegant means for handling these different situations in a unified way, and homogenous coordinates are a way to represent entities (points & lines) in projective spaces.
Vanishing points

Different directions correspond to different vanishing points
Vanishing Points
Beyond the pinhole Camera
Getting more light – Bigger Aperture
Pinhole Camera Images with Variable Aperture

2 mm

1 mm

0.6 mm

0.35 mm

0.15 mm

0.07 mm
The reason for lenses
We need light, but big pinholes cause blur.
Thin Lens

- Rotationally symmetric about optical axis.
- Spherical interfaces.
Thin Lens: Center

- All rays that enter lens along line pointing at \( O \) emerge in same direction.
Thin Lens: Focus

Parallel lines pass through the focus, F
Thin Lens: Image of Point

- All rays passing through lens and starting at $P$ converge upon $P'$
- So light gather capability of lens is given the area of the lens and all the rays focus on $P'$ instead of become blurred like a pinhole
Thin Lens: Image of Point

Relation between depth of Point (-Z) and the depth where it focuses (Z’)

\[
\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}
\]
Thin Lens: Image Plane

A price: Whereas the image of \( P \) is in focus, the image of \( Q \) isn’t.
Thin Lens: Aperture

- Smaller Aperture
  -> Less Blur
- Pinhole
  -> No Blur
Equation of Perspective Projection

Cartesian coordinates:

- We have, by similar triangles, that \((x', y', z') = (f' x/z, f' y/z, f')\)
- Establishing an image plane coordinate system at \(C'\) aligned with \(i\) and \(j\), image coordinates of the projection of \(P\) are \((x, y, z) \rightarrow (f' \frac{x}{z}, f' \frac{y}{z})\)
The equation of projection

Cartesian coordinates:

\[(X, Y, Z) \rightarrow (f \frac{X}{Z}, f \frac{Y}{Z}) = (x, y)\]

Homogenous Coordinates and Camera matrix:

\[
\begin{pmatrix}
  x \\
  y \\
  w
\end{pmatrix}
= \begin{pmatrix}
  f & 0 & 0 & 0 \\
  0 & f & 0 & 0 \\
  0 & 0 & 1 & 0
\end{pmatrix}
\begin{pmatrix}
  X \\
  Y \\
  Z \\
  1
\end{pmatrix}
\]
What if camera coordinate system differs from world coordinate system?

Camera coordinate frame

World coordinate frame

X
Euclidean Coordinate Systems
Coordinate Change: Translation Only

\[ X' = X + t \]
Coordinate Change: Rotation Only

\[ X' = R X \]
Coordinate Changes: Rotation and Translation

\[ X' = R X + t \]
Some points about SO(n)

• \( \text{SO}(n) = \{ R \in \mathbb{R}^{n \times n} : R^T R = I, \det(R) = 1 \} \)
  – \( \text{SO}(2) \): rotation matrices in plane \( \mathbb{R}^2 \)
  – \( \text{SO}(3) \): rotation matrices in 3-space \( \mathbb{R}^3 \)

• Forms a Group under matrix product operation:
  – Identity
  – Inverse
  – Associative
  – Closure

• Closed (finite intersection of closed sets)

• Bounded \( R_{i,j} \in [-1, +1] \)

• Does not form a vector space.

• Manifold of dimension \( n(n-1)/2 \)
  – \( \dim(\text{SO}(2)) = 1 \)
  – \( \dim(\text{SO}(3)) = 3 \)
Parameterizations of SO(3)

– Even though a rotation matrix is 3x3 with nine numbers, it only has three degrees of freedom. It can be parameterized with three numbers. There are many parameterizations.

• Other common parameterizations
  – Euler Angles
  – Axis Angle
  – Quaternions
    • four parameters; homogeneous
Rotation: Homogenous Coordinates

• About z axis

\[
\begin{bmatrix}
x' \\
y' \\
z'
\end{bmatrix} =
\begin{bmatrix}
\cos \theta & -\sin \theta & 0 \\
\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
\]
Rotation

• About x axis:

\[
\begin{pmatrix}
x' \\
y' \\
z'
\end{pmatrix} =
\begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \theta & -\sin \theta \\
0 & \sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix}
\]

• About y axis:

\[
\begin{pmatrix}
x' \\
y' \\
z'
\end{pmatrix} =
\begin{pmatrix}
\cos \theta & 0 & \sin \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix}
\]
Euler Angles: Roll-Pitch-Yaw

• Composition of rotations

\[
R = R_Z(\gamma) \, R_Y(\beta) \, R_X(\alpha)
\]

\[
R = \begin{pmatrix}
  \cos \gamma & -\sin \gamma & 0 \\
  \sin \gamma & \cos \gamma & 0 \\
  0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
  \cos \beta & 0 & \sin \beta \\
  0 & 1 & 0 \\
  -\sin \beta & 0 & \cos \beta
\end{pmatrix}
\begin{pmatrix}
  1 & 0 & 0 \\
  0 & \cos \alpha & -\sin \alpha \\
  0 & \sin \alpha & \cos \alpha
\end{pmatrix}
\]
What if camera coordinate system differs from world coordinate system?

\[
\begin{align*}
X_{\text{Camera}} &= RX_{\text{World}} + t \\
\begin{bmatrix}
X_{\text{Camera}} \\
1
\end{bmatrix} &= \begin{bmatrix}
R & t \\
t^T & 1
\end{bmatrix} \begin{bmatrix}
X_{\text{World}} \\
1
\end{bmatrix}
\end{align*}
\]
Intrinsic parameters

- 3x3 homogenous matrix
- Focal length
- Principal Point
- Units (e.g. pixels)
- Pixel Aspect ratio
Camera Calibration

Given $n$ points $P_1, \ldots, P_n$ with known positions and their images $p_1, \ldots, p_n$, estimate intrinsic and extrinsic camera parameters

- See Text book for how to do it.
- Camera Calibration Toolbox for Matlab (Bouguet)
  - http://www.vision.caltech.edu/bouguetj/calib_doc/
Camera parameters

- Extrinsic Parameters: Since camera may not be at the origin, there is a rigid transformation between the world coordinates and the camera coordinates.
- Intrinsic parameters: Since scene units (e.g., cm) differ image units (e.g., pixels) and coordinate system may not be centered in image, we capture that with a 3x3 transformation comprised of focal length, principal point, pixel aspect ratio, and skew.

\[
\begin{pmatrix}
    x \\
    y \\
    w
\end{pmatrix} = \begin{pmatrix}
    \text{Transformation represented by intrinsic parameters} & \begin{pmatrix}
    1 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 \\
    0 & 0 & 1 & 0
\end{pmatrix} & \text{Rigid Transformation represented by extrinsic parameters}
\begin{pmatrix}
    X \\
    Y \\
    Z \\
    T
\end{pmatrix}
\end{pmatrix}
\]

\[
\begin{pmatrix}
    x \\
    y \\
    w
\end{pmatrix} = \begin{pmatrix}
    \text{3 x 3} \\
    \text{4 x 4}
\end{pmatrix}
\]
Camera Models

- Perspective Projection
- Affine Camera Model
- Scaled Orthographic Projection
- Orthographic Projection

Parallel Projection Camera Models
For all cameras?
Other camera models

- Generalized camera – maps points lying on rays and maps them to points on the image plane.
Some Alternative “Cameras”
Next Lecture

• Image Formation: Light and Shading
• Reading:
  – Chapter 2: Light and Shading