1 Introduction

The purpose of this assignment is three-fold. First, it aims to help you practice the application of Boolean algebra theorems to transform and reduce Boolean expressions. The second goal is to help you learn how to go from the world of Boolean expressions to the world of digital circuits. The final goal is to help you translate a problem described in words to a Boolean algebraic expression. We hope you can think of why each of these exercises is useful when designing digital circuits.

2 Application of Boolean Algebra Theorems

2.1 Prove the DeMorgan’s Law using Boolean algebra:

Prove the DeMorgan’s law, i.e. \((AB)' = A' + B'\).

2.2 Prove the following equations using Boolean algebra:

A. \(a'd + bc'd + abc' = a'd + abc'\).
B. \((a' + d)(b + c' + d)(a + b + c') = (a' + d)(a + b + c')\)

Are the above equations related to the consensus theorem?

2.3 Prove the above two equations using Shannon’s expansion.

3 From Problem to Boolean Expression

A majority voting machine inputs four binary bits \((x_3, x_2, x_1, x_0)\), and outputs \(y = 0\) when more than half of the input bits are 1. Otherwise, the output is \(y = 1\).

i. Write the truth table.
ii. Write the output as a function of the input bits in sum-of-products canonical form.
iii. Write the output as a function of the input bits in product-of-sums canonical form.

4 Boolean Algebra and Implementation

i. Simplify each of the following two Boolean equations.
ii. Sketch a reasonably simple combinational circuit implementing the simplified equation.
iii. Compare the numbers of literals and operators versus the numbers of gates, nets, and pins in the schematic diagrams

A. \(abc'd + ab'c + bc'd + ab'c' + acd + a'bcd\)
B. \((b' + ac')(a'b' + c)(a + b)\)

5  BSV Adder

Complete the following BSV implementation of a full adder (shown above) using \\&, |, and ^ for logic AND, OR, and XOR, respectively. Please refer to BSV by Example for any syntax questions.

function Bit#(2) fa (Bit#(1) a, Bit#(1) b, Bit#(1) c_in);
  Bit#(1) s = ________________________;
  Bit#(1) c_out = _________________________;
  return {c_out, s};
endfunction

6  Assignment Turn-in

Once you’re finished with the assignment, turn it in as a pdf on Gradescope. The codes for two sections are Section A: 92RNVYV and Section B: MN8K74.