Solutions for Midterm1B are provided.

1 True or False

Grading: For each question, 4 points for correct answer; 2 points for wrong answer (wrong choice but reasonable explanation); 0 for making no attempts / wrong choice without explanation

(1) For Boolean algebra, given an equation $xyz = xz$, we can deduce that $y = 1$.
   False. $y = x + z + W$ is a valid choice.

(2) For Boolean algebra, the statement $a + a = a$ is true and can be proven by the laws of Boolean algebra.
   True. $a + a = a$ can be proven true by distributive law and identity law of Boolean algebra.

(3) For a switching function of three inputs, i.e. $f(a, b, c)$ and its corresponding K-map, the number of prime implicants can be 5 or more.
   True. An example is $\Sigma m(0,1,2,5,6,7)$, in which we can find 6 prime implicants.

(4) \{AND,NOT\} is a universal set of gates.
   True. One can use DeMorgan’s law to implement OR from \{AND,NOT\}, so the \{AND,NOT\} is a universal set.

(5) \{NOT,XOR\} is a universal set of gates.
   False. A universal set should be able to implement \{AND,OR,NOT\}; however, AND is unable to be constructed by \{NOT,XOR\}.

(6) \{f(a, b, c) = a'b' + b'c' + a'c'\} is a universal set of gates.
   True. $f$ can implement NOT with $f(a, 0, 1) = a'$, and OR with $f(f(a, b, 1), 0, 1)$; given \{OR,NOT\} is a universal set, $f$ is a universal set of gates.

2 Consensus Theorem Application

Grading:

- no points taken for writing out 0/1 in 2.1/2.2
- 0.5 point taken for any additional wrong term(s)

2.1

(1) 2 points for each term
(2) 1 point for each term, no points for blank
(3) 2 points for each term
(4) 0.5 point for each term, no points for blank

2.2

(1) 2 points for each term
(2) 1 point for each term, no points for blank
(3) 1 point for each term, no points for blank
(4) 0.4 point for each term, no points for blank
(1) $a'b + ac' + bc'$, Consensus(es): $bc'$
(2) $abc'd' + acd'e' + a'be$, Consensus(es): $abd'e'$, $bc'd'e$
(3) $abd + a'cd + bfg$, Consensus(es): $bcd$
(4) $a'b'cd + a'd'e' + bde'$, Consensus(es): $a'be'$, $a'cde'$, $a'be'$

2.2

(1) $(a + b')(a' + c')(b' + c')$, Consensus(es): $(b' + c')$
(2) $(a + c' + e)(a + c + e)(a' + c + d)$, Consensus(es): $(a + e)$, $(c + d + e)$
(3) $(a + b + c + d')(a + c' + d' + e)(a + c' + d)$, Consensus(es): $(a + b + d' + e)$, $(a + c' + e)$
(4) $(a + b + c)(a' + d)(a + b' + d)(a' + c + d')$, Consensus(es): $(b + c + d)$, $(a + c + d)$, $(b + c + d')$, $(b' + d)$, $(a' + c)$

3 Karnaugh Map: Sum of Products Expressions

Grading:
- 20 points for correct answers
- 16-19 points for partially correct switching functions, 1 point taken for each incorrect/missing/extra switching function
- 12 points for wrong K-map, and wrong switching functions (correct derivation)
- 10 points for correct K-map, but no switching functions
- 10 points for correct switching functions without K-map
- 5 points for writing out the correct methodologies without any implementation
- 2 points for writing out the wrong methodologies without any implementation
- 0 point for making no attempt

$f(a, b, c, d) = \Sigma m(0, 1, 2, 5, 7, 9, 12, 14) + \Sigma d(6, 4, 11)$.

The Karnaugh map:
First, we find the essential prime implicants: $\Sigma m(4, 5, 6, 7)$, $\Sigma m(0, 2, 4, 6)$, $\Sigma m(4, 6, 12, 14)$
Then, for the 1s left, we find all possible prime implicants.

For $m(9)$, we have
- $\Sigma m(9, 11)$
- $\Sigma m(1, 9)$

For $m(1)$, we have
- $\Sigma m(0, 1, 4, 5)$

To construct the minimal sum of products expression, $\Sigma m(1, 9)$ is the only choice.

(1) $f = bd' + a'd' + a'b + b'c'd$
4 Karnaugh Map: Product of Sums Expressions

Grading:

- 20 points for correct answers
- 16,18 points for partial correct switching functions, 2 points taken for each wrong switching function (half of the points deducted for correct answer in wrong format)
- 15 points for correct K-map, essential/ prime implicants (correct/wrong), but no switching functions
• 12 points for wrong K-map, and wrong switching functions (correct derivations)
• 10 points for correct K-map, but no switching functions
• 5-9 points for partially correct K-map without switching functions, 1 point taken for each wrong slot,
• 10 points for correct switching functions without K-map
• 5 points for writing out the correct methodologies without any implementation
• 2 points for writing out the wrong methodologies without any implementation
• 0 point for making no attempt

\[ f(a, b, c) = \Sigma m(1, 4, 6) + \Sigma d(3, 5) \]

**Figure 2. Karnaugh map**

First, we find the essential prime implicants: ΠM(0, 2). Then, for the 0s left, we find all possible prime implicants.

For M(7), we have

• ΠM(3, 7)
• ΠM(5, 7)

Thus, we arrive at 2 possible minimal product of sums expressions.

1. \[ f = (a + c)(b' + c') \]
2. \[ f = (a + c)(a' + c') \]
5 Other Types of Gates

Grading:

- 20 points for correct answers
- 3 points taken out for wrong or unsimplified answer; for using Shannon’s expansion, 3 points for each correct term in (3) by applying correct methodology, and 3 points for simplifying each term correctly.
- if setting up incorrect Shannon’s equation (i.e swapped 0 and 1), 5 points are deducted
- 10 points for correct answer, but no derivations / process
- 5 points for writing out the correct methodologies without any implementation
- 2 points for writing out the wrong methodologies without any implementation
- 0 point for making no attempt

\[ f(x, y) = (x' + y) \oplus (x + y') \oplus xy \oplus x'y' \oplus x' \]

By Shannon’s expansion, we arrive at

\[ f(x, y) = x \cdot f(1, y) + x' \cdot f(0, y) \quad (1) \]
\[ = x((0 + y) \oplus (1 + y') \oplus 1y \oplus 0y' \oplus 0) + x'((1 + y) \oplus (0 + y') \oplus 0y \oplus 1y' \oplus 1) \quad (2) \]
\[ = x(y \oplus 1 \oplus y \oplus 0 \oplus 0) + x'(1 \oplus y' \oplus 0 \oplus y' \oplus 1) \quad (3) \]

The properties of XOR that we can use are:

- Associative
- Commutative
- \( x \oplus x = 0 \)
- \( x \oplus x' = 1 \)
- \( x \oplus 0 = x \)
- \( x \oplus 1 = x' \)

Eventually, we have

\[ f(x, y) = x(1) + x'(0) \quad (4) \]
\[ = x \quad (5) \]