Solutions for Midterm1A are provided.

1 True or False

Grading: For each question, 4 points for correct answer; 2 points for wrong answer (wrong choice but reasonable explanation); 0 for making no attempts / wrong choice without explanation

(1) For Boolean algebra, given an equation \( a + b = a \), we can deduce that \( b = 0 \).
   False. \( b = a + X \) is a valid choice.

(2) In Boolean algebra, given the theorem that \( a + 1 = 1 \), we can derive from duality that \( a' \cdot 0 = 0 \) but not \( a \cdot 0 = 0 \).
   False. To perform duality, we swap (+, ·) operators, and interchange all (1, 0), and eventually arrive at \( a \cdot 0 = 0 \).

(3) For a four input switching function, i.e. \( f(a, b, c, d) \) and its corresponding K-map, the number of essential prime implicants may not be larger than 8.
   True. There are 16 slots on the K-map, and at most 8 interleaves.

(4) Shannon’s expansion has \( f(x, y, z) = ((x' + f(0, y, z)) \cdot (x + f(1, y, z))) \).
   False. The correct form of Shannon’s expansion is \( f(x, y, z) = ((x' + f(1, y, z)) \cdot (x + f(0, y, z))) \).

(5) \( \{\text{AND, OR}\} \) is a universal set of gates.
   False. A universal set should be able to implement \( \{\text{AND, OR, NOT}\} \); however, Not is unable to be constructed by \( \{\text{AND, OR}\} \).

(6) \( \{f(a, b) = a' + b'\} \) is a universal set of gates.
   True. \( f(a, 1) \) is NOT; \( f(a', b') = a + b \) is OR; and \( \{\text{OR, NOT}\} \) is a universal set.

2 Consensus Theorem Application

Grading:

- no points taken for writing out 0/1 in 2.1/2.2
- 0.5 point taken for any additional wrong term(s)

2.1

(1) 2 points for each term
(2) 2 points for each term
(3) 2 points for each term
(4) 0.5 point for each term, no points for blank

2.2

(1) 2 points for each term
(2) 2 points for each term, no points for blank
(3) 1 point for each term, no points for blank
(4) 0.5 point for each term, no points for blank

2.1

(1) \( a'b' + a'c + bc \), Consensus(es): \( a'c \)
(2) \( ab'cd + abde' + a'e' + cde' \), Consensus(es): \( acde' \)

(3) \( abcd + a'd + efg \), Consensus(es): \( bcd \)

(4) \( ab'cd + a'de + bde' \), Consensus(es): \( b'cde, acde', a'bd \)

2.2

(1) \( (a' + b')(a' + c')(b + c') \), Consensus(es): \( (a' + c') \)

(2) \( (a + b' + c')(a + c + e)(a' + c + d) \), Consensus(es): \( (a + b' + e), (c + d + e) \)

(3) \( (a + b' + c + d)(a + c + d + e)(a' + c + d) \), Consensus(es): \( (b' + c + d), (c + d + e) \)

(4) \( (a + b + c)(a + d')(a + b' + e)(a' + c + d) \), Consensus(es): \( (a + c + e), (b + c + d), (b' + c + d + e) \)

3 Karnaugh Map: Sum of Products Expressions

Grading:

- 20 points for correct answers
- 14-19 points for partial correct switching functions, 1 point taken for each incorrect/missing/extra switching function
- 12 points for wrong K-map, and wrong switching functions (correct derivation)
- 10 points for correct K-map, but no switching functions
- 10 points for correct switching functions without K-map
- 5 points for writing out the correct methodologies without any implementation
- 2 points for writing out the wrong methodologies without any implementation
- 0 point for making no attempt

\[ f(a, b, c, d) = \Sigma m(0, 1, 3, 5, 7, 8, 12, 14, 15) + \Sigma d(6, 10, 11). \]

The Karnaugh map:
First, we find the essential prime implicants: \( \Sigma m(1, 3, 5, 7), \Sigma m(8, 10, 12, 14) \). Then, for the 1s left, we find all possible prime implicants.

For \( m(0) \), we have
- \( \Sigma m(0, 1) \)
- \( \Sigma m(0, 8) \)

For \( m(15) \), we have
- \( \Sigma m(3, 7, 11, 15) \)
- \( \Sigma m(6, 7, 14, 15) \)
- \( \Sigma m(10, 11, 14, 15) \)

Thus, we arrive at \( 3 \times 2 = 6 \) possible minimal sum of products expressions.

(1) \( f = a'd + ad' + a'b'c + cd \)

(2) \( f = a'd + ad' + b'c'd' + cd \)

(3) \( f = a'd + ad' + a'b'c + bc \)

(4) \( f = a'd + ad' + b'c'd' + bc \)

(5) \( f = a'd + ad' + a'b'c' + ac \)

(6) \( f = a'd + ad' + b'c'd' + ac \)
Figure 1. Karnaugh map

4 Karnaugh Map: Product of Sums Expressions

Grading:
- 20 points for correct answers
- 16, 18 points for partial correct switching functions, 2 points taken for each wrong switching function (half of the points deducted for correct answer in wrong format)
- 15 points for correct K-map, essential/ prime implicants (correct/wrong), but no switching functions
- 12 points for wrong K-map, and wrong switching functions (correct derivations)
- 10 points for correct K-map, but no switching functions
- 5-9 points for partially correct K-map without switching functions, 1 point taken for each wrong slot,
• 10 points for correct switching functions without K-map
• 5 points for writing out the correct methodologies without any implementation
• 2 points for writing out the wrong methodologies without any implementation
• 0 point for making no attempt

\[ f(a, b, c) = \Sigma m(0, 2, 5) + \Sigma d(3, 6) \]

First, we find the essential prime implicants: \( \Pi M(1, 3), \Pi M(4, 6) \). Then, for the 0s left, we find all possible prime implicants.

For \( M(7) \), we have
- \( \Pi M(3, 7) \)
- \( \Pi M(6, 7) \)

Thus, we arrive at 2 possible minimal product of sums expressions.

1. \( f = (a' + c)(a + c')(b' + c') \)
2. \( f = (a' + c)(a + c')(a' + b') \)

5 Other Types of Gates

Grading:
• 20 points for correct answers
• 3 points taken out for wrong or unsimplified answer; for using Shannon’s expansion, 3 points for each correct term in (3) by applying correct methodology, and 3 points for simplifying each term correctly.
• if setting up incorrect Shannon’s equation (i.e swapped 0 and 1), 5 points are deducted
• 10 points for correct answer, but no derivations / process
• 5 points for writing out the correct methodologies without any implementation
• 2 points for writing out the wrong methodologies without any implementation
• 0 point for making no attempt

\[ f(x, y) = x'y \oplus xy' \oplus (x + y) \oplus (x' + y') \oplus y' \]

By Shannon’s expansion, we arrive at

\[ f(x, y) = xf(1, y) + x'f(0, y) \]
\[ = x(0y \oplus 1y' \oplus (1 + y) \oplus (0 + y') \oplus y') + x'(1y \oplus 0y' \oplus (0 + y) \oplus (1 + y') \oplus y') \]
\[ = x(0 \oplus y' \oplus 1 \oplus y' \oplus y') + x'(y \oplus 0 \oplus y \oplus 1 \oplus y') \]

The properties of XOR that we can use are:

• Associative
• Commutative
• \( x \oplus x = 0 \)
• \( x \oplus x' = 1 \)
• \( x \oplus 0 = x \)
• \( x \oplus 1 = x' \)

Eventually, we have

\[ f(x, y) = x(y) + x'(y) \]
\[ = y \]