CSE 140 Discussion 1

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HW1: Introduction

• The purpose of this assignment is three-fold:
  – Practice the application of Boolean algebra theorems to transform and reduce Boolean expressions.
  – Go from the world of Boolean expressions to the world of digital circuits.
  – Translate a problem described in words to a Boolean algebraic expression.
Problem 2.1

• Prove DeMorgan’s Law using Boolean algebra
  – \((AB)' = A' + B'\)
  – \((A + B)' = A'B'\)
  – Hint: To prove LHS' = RHS, show RHS is complement of LHS, i.e.,
    • LHS.RHS = 0, and
    • LHS + RHS = 1.
Problem 2.2

• Consensus Theorem
  – \( AB + BC + A’C = AB + A’C \)
  – \( (A + B)(B + C)(A’ + C) = (A + B)(A’ + C) \)
  – Use Boolean algebra to prove
    • Hint: same steps as in proving consensus theorem.
Problem 2.3

• Shannon’s Expansion:
  – \( f(a,b,c) = a \cdot f(a=1,b,c) + a' \cdot f(a=0,b,c) \)
  – \( f(a,b,c) = (a + f(a=0,b,c)) \cdot (a' + f(a=1,b,c)) \)
  – Which variable to choose?
    • Hint: Consensus theorem
Problem 3

• Problem (English) to truth table
• Truth table to Boolean algebra
  – Sum of products
  – Product of sums
• Example:
  – 3-bit input: x1, x2, x3
  – Output $y = 1$ when number of 1’s in input is odd and $y = 0$ otherwise.
### Example

<table>
<thead>
<tr>
<th>x1</th>
<th>x2</th>
<th>x3</th>
<th>y</th>
<th>minterm</th>
<th>MAXterm</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$x'1.x'2.x'3$</td>
<td>$x1+x2+x3$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>$x'1.x'2.x3$</td>
<td>$x1+x2+x'3$</td>
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</table>

SOP: \( y = x'1.x'2.x3 + x'1.x2.x'3 + x1.x'2.x'3 + x1.x2.x3 \)

POS: \( y = (x_1+x_2+x_3).(x_1+x'2+x'3).(x'1+x_2+x'3).(x'1+x'2+x3) \)
Problem 4

- Useful tools for simplification:
- $a + ab = a$ (Absorption)
- $a + a'b$
  - add consensus: $a + a'b + b$
  - absorption: $a + b$
- $ad + bcd + a'c$
  - add consensus: $ad + bcd + a'c + cd$
  - absorption: $ad + a'c + cd$
  - remove consensus: $ad + a'c$
\[ ad + b'd + a'bc' + a'b'c + abc \]

- **#Variables:**
  - 4
- **#Literals:**
  - 13
- **#Operators:**
  - 6
- **#Gates:**
  - 6
- **#Nets:**
  - 10
- **#Pins:**
  - 24

texts:

- nets: #variables + #operators
- pins: #literals + gates*2 - 1
BSV Demo

• Problem 5
  – Test your code
• Problem 4
  – Test function equivalence