The first exam for this class is on Wednesday April 25. The exam covers Chapters 0 and 1 of Sipser, which is up to and including the lecture on Monday, April 23.

1. Let \( L \) be the language over the alphabet \( \{0, 1\} \) defined by

\[
L = \{w \mid \text{w contains an even number of 0's and an odd number of 1's and does not contain the substring 01}\}.
\]

Give a DFA with at most five states that recognizes \( L \).

[[ Optional extra practice: (1) Is there an NFA with fewer states that also recognizes \( L \)? (2) Give a regular expression that describes \( L \). ]]

2. Consider the NFA \( N \) over the alphabet \( \{a, b, c\} \) with the state diagram shown below.

(a) Which of the following strings are accepted by \( N \)?
   i. abc
   ii. cbbc
   iii. cbbea
   iv. \( \varepsilon \)

(b) Write the formal definition for \( N \).

[[ Optional extra practice: (1) Find a DFA that recognizes \( L(N) \). (2) Write a regular expression for \( L(N) \). ]]

3. Give the setup and construction steps of a proof that shows that the class of regular languages over an alphabet \( \Sigma \) is closed under the operation \( \text{EvenLengthStringsOnly}(L) \), defined as

\[
\text{EvenLengthStringsOnly}(L) = \{w \in L \text{ such that } |w| \text{ is even}\}.
\]

Show how your general construction works on the example language of all binary strings containing the substring 101.
4. **True or False** Briefly justify each answer.

(a) For every DFA or NFA, $M$, over $\Sigma$, $L(M) = \Sigma^*$ if and only if each state is an accept state.

(b) Whenever $R_1$ is a regular expression over the alphabet $\{a, b, c\}$, $L((R_1 \circ \emptyset) \circ c) = L((R_1 \circ \varepsilon) \circ c)$.

(c) In a proof that a language is not regular using the Pumping Lemma, we should never choose $i = 1$. (Using the standard variables from the textbooks and class where $s$ is the string, $s = xyz$, and $i$ is the number of times to repeat $y$.)

(d) For all sets $A$, $B$, if $A$ and $B$ are both nonregular then $A \cap B$ is also nonregular.

(e) For all sets $L$, $L$ is regular if and only if $L^*$ is regular.