Today's learning goals

- Design NFA recognizing a given language
- Convert a NFA (with or without spontaneous moves) to a DFA recognizing the same language
NFA

$L(M1) = \neg \epsilon$

$L(M2) = \neg \epsilon$
NFA

Formal definition of M1:

Formal definition of M2:
NFA

Concatenation?
Union?
Simulating NFA with DFA

Not quite a closure proof, but …

Proof:

Given name variables for sets, machines assumed to exist.

WTS state goal and outline plan.

Construction using objects previously defined + new tools working towards goal. Give formal definition and explain.

Correctness prove that construction works.

Conclusion recap what you've proved.
Simulating NFA with DFA

For any language recognized by an NFA, there is a DFA that recognizes this language.

Proof:

Given $A$, a language recognized by $N = (Q, \Sigma, \delta, q_0, F)$ a NFA, WTS there is some DFA $M$ with $L(M) = A$

Construction

Correctness

Conclusion
Idea of construction

Track set of possible states NFA might be in.
Which states can this NFA be in before first input symbol is read?

A. q0
B. any state
C. q0, q1
D. q0, q4
E. q0, q1, q4
Subset construction

Given $A$, a language recognized by $N = (Q, \Sigma, \delta, q_0, F)$ a NFA

WTS there is some DFA $M$ with $L(M) = A$

Construction Define $M = (Q', \Sigma, \delta', q_0', F')$ with

- $Q' = \text{the power set of } Q = \{ X \mid X \text{ is a subset of } Q \}$
- $q_0' = \{ \text{states } N \text{ can be in before first input symbol read} \}$
- $F' = \{ \}$
- $\delta' ( \quad ) =$
Subset construction

Given $A$, a language recognized by $N = (Q, \Sigma, \delta, q_0, F)$ a NFA

WTS there is some DFA $M$ with $L(M) = A$

Construction Define $M = (Q', \Sigma, \delta', q_0', F')$ with

- $Q' = \text{the power set of } Q = \{ X \mid X \text{ is a subset of } Q \}$
- $q_0' = \{ q_0 \} \cup \delta((q_0, \epsilon))$ …
- $F' = \{ \ldots \}$
- $\delta' (\ldots) =$
Subset construction

Given $A$, a language recognized by $N = (Q, \Sigma, \delta, q_0, F)$ a NFA

WTS there is some DFA $M$ with $L(M) = A$

Construction Define $M = (Q', \Sigma, \delta', q_0', F')$ with

- $Q' = \text{the power set of } Q = \{ X \mid X \text{ is a subset of } Q \}$
- $q_0' = \{ q_0 \} \cup \delta((q_0, \varepsilon))$ ...
- $F' = \{ \ldots \}$
- $\delta' ((X, x)) = \{ q \in Q \mid q \text{ is in } \delta((r, x)) \text{ for some } r \in X \text{ or is accessible via spontaneous moves} \}$

Types?
From NFA to DFA
Subset construction

Given A, a language recognized by $N = (Q, \Sigma, \delta, q_0, F)$ a NFA

WTS there is some DFA $M$ with $L(M) = A$

Construction Define $M = (Q', \Sigma, \delta', q_0', F')$ with

- $Q' = \text{the power set of } Q = \{ X \mid X \text{ is a subset of } Q \}$
- $q_0' = \{ q_0 \} \cup \delta((q_0, \varepsilon))$
- $F' = \{ \text{guarantee at least one computation is successful} \}$
- $\delta' ((X, x)) = \{ q \in Q \mid q \text{ is in } \delta((r,x)) \text{ for some } r \in X \text{ or is accessible via spontaneous moves} \}$
What does it mean for a set of states $X$ to guarantee at least one computation is successful?

A. $X$ is a subset of $F$
B. $X = F$
C. $X \cap F$ is nonempty
D. $X$ is an element of $F$
E. None of the above.
Subset construction examples
DFA equiv NFA

Theorem: For each language $L$, $L$ is recognizable by some DFA iff $L$ is recognizable by some NFA
Theorem: For each language $L$, $L$ is recognizable by some DFA iff $L$ is recognizable by some NFA iff $L$ is describable by some regular expression.
For next time

- Work on Individual HW2  due Tuesday

Pre class-reading for Monday: Example 1.56 on page 68