1. (10 points) Extension of Exam 2 In the exam, we considered the following construction: Suppose $D = (Q, \Sigma, \delta, q_0, F)$ is a DFA. We will define a push-down automaton $D' = (Q \times \{0,1\} \cup \{q'_0, q'_{acc}\}, \Sigma, \Sigma \cup \{$, $\delta'$, $q'_0, \{q'_{acc}\}$) where we assume $q'_0 \notin Q, q'_{acc} \notin Q, \notin \Sigma$ and where $\delta'$ is as follows:

$$
\delta'( (q_0, 0, \varepsilon, \varepsilon) ) = \{(q_0, 0), \$\} \\
\delta'( (q, 0, x, \varepsilon) ) = \{(\delta((q, x), 0), x)\} \quad \text{if } q \in Q, x \in \Sigma \\
\delta'( (q, 0, \varepsilon, \varepsilon) ) = \{(q_1, 1), \varepsilon\} \quad \text{if } q \in F \\
\delta'( (q, 1, x, x) ) = \{(\delta((q, x), 1), \varepsilon\} \quad \text{if } q \in Q, x \in \Sigma \\
\delta'( (q, 1, \varepsilon, \$) ) = \{(q'_{acc}, \varepsilon)\} \quad \text{if } q \in F \\
\delta'( (q, x, y) ) = 0 \quad \text{otherwise}
$$

For each of the following, answer “Yes”, “No”, or “Depends on D’”. For reference, the correct answers (without justifications) to the exam questions were:

Is $\varepsilon \in L(D')$? Depends on D. Is $L(D')$ decidable? Yes.

a. Is $L(D')$ regular?

b. Is $L(D')$ context-free?

c. Is $L(D')$ decidable?

d. Is $L(D') = DOUBLE(L(D))$?

e. Is $L(D') = \{ww^R \mid w \in L(D)\}$?

No justifications required for credit for this question; but, as always, they’re a good idea for your own benefit.
2. (10 points). Let
\[ L_1 = \{ a^i b^j \mid 0 \leq j < i \} \]
\[ L_2 = \{ a^i b^j \mid 0 \leq i < j \} \]

a. Consider the enumerator \( E_1 \) given by the high-level description
\[ E_1 = \text{"Ignore the input}
1. For integer \( i = 0, 1, 2, \ldots \)
2. For integer \( j = 0, 1, 2, \ldots \)
3. If \( j > i \), print \( a^i b^j \)
4. Increment \( j \)
5. Increment \( i \)"

– List the first three strings printed by \( E_1 \).
– Is \( L(E_1) = L_1 \)?
– Is \( L(E_1) = L_2 \)?

b. Consider the enumerator \( E_2 \) given by the high-level description
\[ E_2 = \text{"Ignore the input}
1. For integer \( i = 1, 2, \ldots \)
2. For integer \( j = 0, 1, 2, \ldots, i-1 \)
3. Print \( a^i b^j \)
4. Increment \( j \)
5. Increment \( i \)"

– List the first three strings printed by \( E_2 \).
– Is \( L(E_2) = L_1 \)?
– Is \( L(E_2) = L_2 \)?

No justifications required for credit for this question; but, as always, they’re a good idea for your own benefit.

3. (10 points) Consider the following computational problems over a fixed alphabet \( \Sigma \).
\[ A_{DFA} = \{ \langle A, w \rangle \mid A \text{ is a DFA over } \Sigma, w \in \Sigma^*, w \in L(A) \} \]
\[ E_{DFA} = \{ \langle A \rangle \mid A \text{ is a DFA over } \Sigma, L(A) = \emptyset \} \]
\[ EQ_{DFA} = \{ \langle A, B \rangle \mid A \text{ and } B \text{ are DFAs over } \Sigma \text{ and } L(A) = L(B) \} \]
\[ SUB_{DFA} = \{ \langle A, B \rangle \mid A \text{ and } B \text{ are DFAs over } \Sigma \text{ and } L(A) \subseteq L(B) \} \]
\[ ALL_{DFA} = \{ \langle A \rangle \mid A \text{ is a DFA over } \Sigma, L(A) = \Sigma^* \} \]
\[ INF_{DFA} = \{ \langle A \rangle \mid A \text{ is a DFA over } \Sigma, L(A) \text{ is (countably) infinite} \} \]

a. Find all subset relations between distinct sets in this list. That is, determine when \( ??_{DFA} \subseteq ??_{DFA} \).
b. Find all pairs of sets in this list that are not disjoint. That is, determine when \( ??_{DFA} \cap ??_{DFA} \neq \emptyset \).

No justifications required for credit for this question; but, as always, they’re a good idea for your own benefit.