1. **(10 points)** Find examples of languages satisfying each of the following properties (and briefly justify), or prove that no such language exists.
   a. An infinite recognizable language that is undecidable and whose complement is finite.
   b. A non-context free language that is undecidable.
   c. A decidable language with an undecidable subset.
   d. A language all of whose subsets are recognizable.

2. **(10 points)**
   a. Given a Turing machine $M$, define (using a high-level description) an enumerator $E$ such that
      \[
      L(E) = \{w \mid w \in L(M) \text{ and } w^R \in L(M)\}
      \]
      You do not need to include a formal proof of correctness, but briefly justify the idea of your construction.
   b. Given an enumerator $E$, define (using a high-level description) a Turing machine $M$ such that
      \[
      L(M) = \{w \mid w \in L(E) \text{ and } w^R \in L(E)\}
      \]
      You do not need to include a formal proof of correctness, but briefly justify the idea of your construction.
   c. Referring to your construction in part b, is $M$ a decider? Why or why not?
      You do not need to include a formal proof of correctness, but briefly justify the idea of your construction.
3. (10 points)
   a. Let $\Sigma = \{0, 1\}$. Prove that
      \[ \{ \langle A \rangle \mid A \text{ is a DFA over } \Sigma \text{ and } L(A) \subseteq L(0^*1^*) \} \]
      is decidable.
   b. Recall that
      \[ INF_{DFA} = \{ \langle A \rangle \mid A \text{ is a DFA over } \Sigma, L(A) \text{ is (countably) infinite} \} \]
      Prove that $INF_{DFA}$ is decidable. *Hint: you might find ideas from the Pumping Lemma useful.*