1. (a) Let $A = \{0, 1\} \times \{0, 1\}$ and $B = \{a, b, c\}$. Suppose $A$ is listed in lexicographic order based on $0 < 1$ and $B$ is in alphabetic order. If the set $A \times B \times A$ is listed in lexicographic order, then the next element after $((1, 0), c, (1, 1))$ is

(0) $((1, 0), a, (0, 0))$
(1) $((1, 1), c, (0, 0))$
(2) $((1, 1), a, (0, 0))$
(3) $((1, 1), a, (1, 1))$
(4) $((1, 1), b, (1, 1))$

(b) In each case below, some information about a function is given to you. Answer the following questions and give reasons for your answers:

(i) Have you been given enough information to specify the function?

(ii) Can you tell whether or not the function is an injection? a surjection? a bijection? If so, what is it?

(0) $f \in \{a\}, \text{Coimage}(f) = \{\{1, 3, 5\}, \{2, 4\}\}$. 
(1) $f \in \{a\}, \text{Coimage}(f) = \{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}\}$. 
(2) $f \in \{a\}, f^{-1}(2) = \{1, 3, 5\}, f^{-1}(4) = \{2, 4\}$. 
(3) $f \in \{a\}, |\text{Image}(f)| = 4$.

(c) Let $f : X \to Y$ be a function from $X$ to $Y$. Consider the following statement: For all subsets $C$ and $D$ of $Y$, $f^{-1}(C \cap D^c) = f^{-1}(C) \cap [f^{-1}(D)]^c$. This statement is

(0) True and equivalent to: For all subsets $C$ and $D$ of $Y$,

$$f^{-1}(C - D) = f^{-1}(C) - f^{-1}(D)$$

(1) False and equivalent to: For all subsets $C$ and $D$ of $Y$,

$$f^{-1}(C - D) = f^{-1}(C) - f^{-1}(D)$$

(2) True and equivalent to: For all subsets $C$ and $D$ of $Y$,

$$f^{-1}(C - D) = f^{-1}(C) - [f^{-1}(D)]^c$$

(3) False and equivalent to: For all subsets $C$ and $D$ of $Y$,

$$f^{-1}(C - D) = [f^{-1}(C)]^c - f^{-1}(D)$$

(4) True and equivalent to: For all subsets $C$ and $D$ of $Y$,

$$f^{-1}(C - D) = [f^{-1}(C)]^c - f^{-1}(D)$$

In parts (a) and (c), you must not only indicate the correct answer, but also explain in detail how you arrived at this result.
2. Are the following statements true? If so, prove them. If not, provide a counter-example.

(a) For all sets $A$, $B$, and $C$, if $A \subseteq B$ and $B \cap C \neq \emptyset$, then $A \cap C \neq \emptyset$.
(b) For all sets $A$, $B$, and $C$, if $(A \cap C) \subseteq B$ then $(C - B) \cap (A - B) = \emptyset$.

Using the algebraic rules for sets, simplify the following set expressions.

(c) $(A - B) \cup (A \cap B)$
(d) $(A - (A \cap B)) \cup (A \cap B)$

3. Write a program (in C or in any other language) that prints the first 64 rows of the Pascal triangle modulo 2. Specifically, the entry in row $n$ and column $k$ of the printout should be 0 if $\binom{n}{k}$ is even, and 1 otherwise. Submit both the program and the printout it produces.

4. Let $A$ be a set with $n$ elements. Find an expression for $S(n, 2)$, the number of partitions of $A$ into exactly two subsets. You can either start with the general recurrence for $S(n, k)$, or simply count $S(n, 2)$ directly.

5. Let $A = \{4, 5, 6\}$ and $B = \{5, 6, 7\}$. Consider three binary relations $\mathcal{R}, \mathcal{S}, \mathcal{T}$ from $A$ to $B$, defined as follows:

\[ \forall (a, b) \in A \times B, \quad (a, b) \in \mathcal{R} \iff a \geq b \]
\[ \forall (a, b) \in A \times B, \quad a \mathcal{S} b \iff 2 \mid (a - b) \]
\[ \mathcal{T} = \{(4, 7), (6, 5), (6, 7)\} \]

For each of the three relations $\mathcal{R}, \mathcal{S}, \mathcal{T}$, explain whether or not it is a functional relation.

6. Given any two functions $f: X \to Y$ and $g: Y \to Z$, the composition function $h = g \circ f$ is a function from $X$ to $Z$ defined as follows $h(x) = g(f(x))$ for all $x \in X$.

(a) Suppose $h$ is an injection. Does it follow from this that $f$ is an injection? Does it follow from this that $g$ is an injection?

(b) Suppose $h$ is a surjection. Does it follow from this that $f$ is a surjection? Does it follow from this that $g$ is a surjection?

(c) Suppose $h$ is a bijection. Does it follow from this that $f$ is a bijection? Does it follow from this that $g$ is a bijection?