1. (a) Let \( p, q, \) and \( r \) denote the following statements:
\[
\begin{align*}
p &= \text{“computer is out of memory (COM) flag is off”} \\
q &= \text{“disk error has occurred (DEO) flag is off”} \\
r &= \text{“ZIP disk has not enough memory left (ZIP) flag is off”}
\end{align*}
\]
Write the five statements below in symbolic form.

a. COM is off and DEO is off and ZIP is off.

b. COM is off but DEO is on.

c. There is enough memory in the computer. However, either a disk error has occurred or the ZIP disk is out of memory.

d. The computer is out of memory and no disk error has occurred, but the ZIP disk is out of memory.

e. Either the computer is out of memory or both COM and DEO are off.

(b) Write down a truth table for the statement form \( (p \lor (\sim p \lor q)) \land \sim (q \land \sim r) \).

(c) The truth table for \( (p \lor (r \lor q)) \land (\sim q \land \sim r) \) is the same as the truth table for
\[
\begin{align*}
(0) & \quad q \lor r \\
(1) & \quad ((p \lor r) \lor q) \land (p \lor r) \\
(2) & \quad (p \land q) \lor (p \land r) \\
(3) & \quad (p \lor q) \land \sim (p \lor r) \\
(4) & \quad (p \land r) \lor (p \land q)
\end{align*}
\]

(d) Which of the following statement forms is a tautology?
\[
\begin{align*}
(0) & \quad \sim p \lor (p \land q) \\
(1) & \quad (p \land q) \lor (\sim p \lor (p \land \sim q)) \\
(2) & \quad (p \land \sim q) \land (\sim p \lor q) \\
(3) & \quad ((\sim p \land q) \land (q \land r)) \land \sim q \\
(4) & \quad (\sim p \lor q) \lor (p \land q)
\end{align*}
\]

For full credit, you must not only indicate the correct answer, but also explain in detail how you arrived at this result.
2. The Boolean function \( f(p, q, r) \) is specified by the following truth table:

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
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<th>( f )</th>
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</table>

(a) Write down the disjunctive normal form (DNF) for \( f(p, q, r) \).
(b) Specify \( f(p, q, r) \) in terms of a simpler statement form.
(c) Design a logic circuit implementing this truth table.

3. The exclusive or (XOR) of \( n \) binary variables is 1 if an odd number of the variables is 1, and 0 if an even number of the variables is 1. Design a logic circuit that computes the XOR of \( n \) variables. You can use any two-input gates you want, including two-input XOR’s.

4. The light in your dorm is controlled by three switches. One next to your bed, another next to your roommate’s bed, and a master switch at the dorm master’s office, which enables him to shut down the light in your dorm if he wants to. If the master switch is off, then the light is off. If the master switch is on, then if the two switches in your room are both on or both off the light is off, else it is on. Design a logic circuit that controls the lights.

5. You and your favorite CSE20 classmate are invited to a costume party over Spring break. You ask her how she will dress up as, and she replies:

- If I do not come as Xena and I don’t come as Austin Powers, I’ll dress up as G.I. Jane.
- I will either be at the entrance at 8 p.m or (inclusive) I won’t show up as G.I. Jane.
- If I come as Austin Powers, Madonna will show up too.

You arrive at the party at 8 p.m and: She is not at the entrance and Madonna is nowhere to be seen! (Hello?! This is UCSD.)

(a) Express the three statements your friend made in propositional logic, using the following notation:

- \( x \) - your friend shows up as Xena,
- \( a \) - your friend shows up as Austin Powers,
- \( g \) - your friend shows up as G.I. Jane,
- \( p \) - your friend is at the entrance at 8 pm,
- \( m \) - Madonna shows up at the party.

You may use the implication connective (\( \rightarrow \)) to do so.
(b) Express each of the three statements using only the connectives \( \lor, \land, \) and \( \neg \).
(c) Who did your friend show up as?

6. Are the statement forms \( (p \leftrightarrow q) \rightarrow r \) and \( p \leftrightarrow (q \rightarrow r) \) logically equivalent?