Announcements

- Homework 1 is due today by 11:59 PM
- Homework 2 will be assigned today
  - Due Wed, Apr 26, 11:59 PM
  - To be completed individually, not in groups
- Reading:
  - Section 2.2.4: Photometric Stereo
    - Shape from Multiple Shaded Images

Photometric Stereo
Introduction to Computer Vision
CSE 152
Lecture 4

Shading reveals 3-D surface geometry

Two shape-from-X methods that use shading

- Shape-from-shading: Use just one image to recover shape. Requires knowledge of light source direction and BRDF everywhere. Too restrictive to be useful.

- Photometric stereo: Single viewpoint, multiple images under different lighting.

Photometric Stereo Rigs:
One viewpoint, changing lighting

An example of photometric stereo

- surface (albedo texture map)
- + surface normals
- \( + \) albedo
Photometric stereo

- Single viewpoint, multiple images under different lighting.
  1. Arbitrary known BRDF, known lighting
  2. Lambertian BRDF, known lighting
  3. Lambertian BRDF, unknown lighting.

BRDF

- Bi-directional Reflectance Distribution Function
  \( \rho(\theta_{\text{in}}, \phi_{\text{in}}; \theta_{\text{out}}, \phi_{\text{out}}) \)

- Function of
  - Incoming light direction: \( \theta_{\text{in}}, \phi_{\text{in}} \)
  - Outgoing light direction: \( \theta_{\text{out}}, \phi_{\text{out}} \)

- Ratio of incident irradiance to emitted radiance

Coordinate system

Gradient Space (p,q)

Image Formation

For a given point A on the surface a, the image irradiance \( E(x,y) \) is a function of

1. The BRDF at A
2. The surface normal at A
3. The direction of the light source
Reflectance Map

Let the BRDF be the same at all points on the surface, and let the light direction \( \mathbf{a} \) be a constant.
1. Then image irradiance is a function of only the direction of the surface normal.
2. In gradient space, we have \( E(p,q) \).

Example Reflectance Map: Lambertian surface

Light Source Direction, expressed in gradient space.

What does the intensity (irradiance) of one pixel in one image tell us?
- It constrains the surface normal projecting to that point to a curve

Two Light Sources
Two reflectance maps

A third image would disambiguate match

Three Source Photometric stereo:
Step 1

Offline:
Using source directions & BRDF, construct reflectance map for each light source direction. \( R_1(p,q), R_2(p,q), R_3(p,q) \)

Online:
1. Acquire three images with known light source directions. \( E_1(x,y), E_2(x,y), E_3(x,y) \)
2. For each pixel location \((x,y)\), find \((p,q)\) as the intersection of the three curves
   \[ R_1(p,q) = E_1(x,y) \]
   \[ R_2(p,q) = E_2(x,y) \]
   \[ R_3(p,q) = E_3(x,y) \]
3. This is the surface normal at pixel \((x,y)\). Over image, the normal field is estimated
Normal Field

Plastic Baby Doll: Normal Field

Next step:
Go from normal field to surface

Recovering the surface $f(x,y)$

Many methods: Simplest approach
1. From estimate $n = (n_x, n_y, n_z)$, $p = n_x/n_z$, $q = n_y/n_z$
2. Integrate $p = df/dx$ along a row $(x,0)$ to get $f(x,0)$
3. Then integrate $q = df/dy$ along each column starting with value of the first row

What might go wrong?

• Height $z(x,y)$ is obtained by integration along a curve from $(x_0, y_0)$.

$$z(x,y) = z(x_0,y_0) + \int_{x_0}^{x} (pdx + qdy)$$

• If one integrates the derivative field along any closed curve, one expects to get back to the starting value.

• Might not happen because of noisy estimates of $(p,q)$

Integrability. If $f(x,y)$ is the height function, we expect that

$$\frac{\partial}{\partial y} \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} \frac{\partial f}{\partial y}$$

In terms of estimated gradient space $(p,q)$, this means:

$$\frac{\partial p}{\partial y} = \frac{\partial q}{\partial x}$$

But since $p$ and $q$ were estimated independently at each point as intersection of curves on three reflectance maps, equality is not going to exactly hold.
Horn’s Method
[ "Robot Vision, B.K.P. Horn, 1986 ]

- Formulate estimation of surface height \( z(x, y) \) from gradient field by minimizing cost functional:

\[
\int_{\text{Image}} (z_x - p)^2 + (z_y - q)^2 \, dx \, dy
\]

where \((p, q)\) are estimated components of the gradient while \( z_x \) and \( z_y \) are partial derivatives of best fit surface

- Solved using calculus of variations – iterative updating

- \( z(x, y) \) can be discrete or represented in terms of basis functions.

- Integrability is naturally satisfied.

2. Photometric Stereo:
Lambertian Surface, Known Lighting

Lambertian Surface

At image location \((u, v)\), the intensity of a pixel \( x(u, v) \) is:

\[
e(u, v) = [a(u, v) \cdot n(u, v)] \cdot [s_0 \cdot s] = b(u, v) \cdot s
\]

where
- \( a(u, v) \) is the albedo of the surface projecting to \((u, v)\).
- \( n(u, v) \) is the direction of the surface normal.
- \( s_0 \) is the light source intensity.
- \( s \) is the direction to the light source.

What if we have more than 3 Images?
Linear Least Squares

\[
[e_1, e_2, e_3, \ldots, e_n] = b^T[s_1, s_2, s_3, \ldots, s_n]
\]

Let the residual be

\[
\mathbf{r}^T = \mathbf{e}^T - b^T \mathbf{s}
\]

Squaring this:

\[
\mathbf{r}^T \mathbf{r} = (\mathbf{e} - b \mathbf{s})^T (\mathbf{e} - b \mathbf{s})
\]

\[
= \mathbf{e}^T \mathbf{e} - 2 \mathbf{b}^T \mathbf{S}^T \mathbf{e} + \mathbf{b}^T \mathbf{S} \mathbf{S} \mathbf{b}
\]

\[
\frac{\partial \mathbf{r}^T \mathbf{r}}{\partial \mathbf{b}} = 0
\]

- zero derivative is a necessary condition for a minimum, or

\[
-2 \mathbf{S}^T \mathbf{e} + 2 \mathbf{S} \mathbf{S} \mathbf{b} = 0;
\]

\[
\mathbf{b} = (\mathbf{S} \mathbf{S})^{-1} \mathbf{S}^T \mathbf{e}
\]

Input Images
Recovered albedo

Recovered normal field

Surface recovered by integration

An example of photometric stereo

Next Lecture

- Binary image processing
- Reading:
  - Szeliski, Chapter 3 Image processing, Section 3.3 More neighborhood operators