CSE 140: Components and Design Techniques for Digital Systems

Lecture 6: Universal Gates

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Combinational Logic: Various Types of Gates

- Universal Set of Gates
  - Motivation
  - Definition
  - Examples
- Other Types of Gates
  - XOR
  - NAND / NOR
  - Block Diagram Transfers

Universal Set of Gates: Motivation

- AND, OR, NOT: Logic gates related to reasoning from Aristotle (384-322BCE)
- NAND, NOR: Inverted AND, Inverted OR gates. For VLSI technologies, all gates are inverted (AND,OR operation with a bubble at output).
- Multiplexer + input table: Table based logic for programmability. FPGA technology.
- XOR: Exclusive OR gates. Parity check.
- Neuron and Synapse: Neural network

In the future, we may have new sets of gates due to new technologies. Given a set of gates, can the gates in the set cover all possible switching functions?

Universal Set

Universal set is a powerful concept to identify the coverage of a set of gates afforded by a given technology. Criterion: If the set of gates can implement AND, OR, and NOT gates, the set is universal.

Universal Set Definition

Universal Set: A set of gates such that every switching function can be implemented with the gates in this set.

Examples

{AND, OR, NOT}

{AND, NOT}

{OR, NOT}

Universal Set: Examples

Universal Set: A set of gates such that every switching function can be implemented with the gates in this set.

Examples

{AND, OR, NOT}

{AND, NOT} OR can be implemented with AND & NOT gates: $a+b = (a' b')'$

{OR, NOT} AND can be implemented with OR & NOT gates: $ab = (a' + b')'$

{XOR} is not universal

{XOR, AND} is universal

iClicker

Is the set {AND, OR} (but no NOT gate) universal?

A. Yes

B. No

iClicker

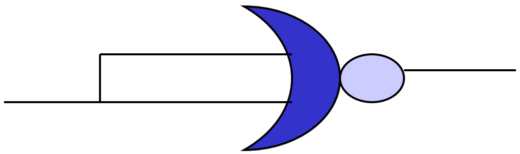
Is the set $\{f(x,y)=xy'\}$ universal?

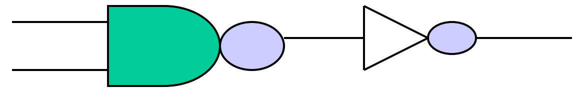
A. Yes

B. No

Universal Set: Examples

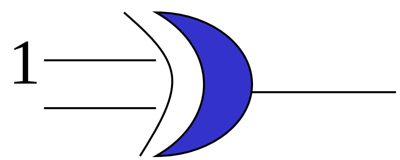
{NAND, NOR}





{XOR}

{XOR, AND}



$X \oplus 1 = X*1' + X'*1 = X'$ if constant "1" is available.

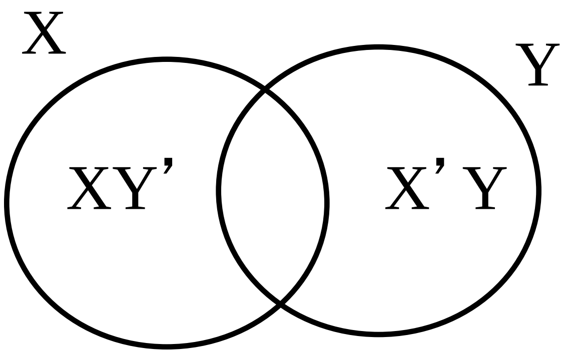
Other Types of Gates: Properties and Usage

1) XOR $X \oplus Y = XY' + X'Y$
2) NAND, NOR
3) Block Diagram Transfers

Other Types of Gates: XOR

1) $X \oplus Y = XY' + X'Y$

It is a parity function (examples) useful for testing because the flipping of a single input changes the output.

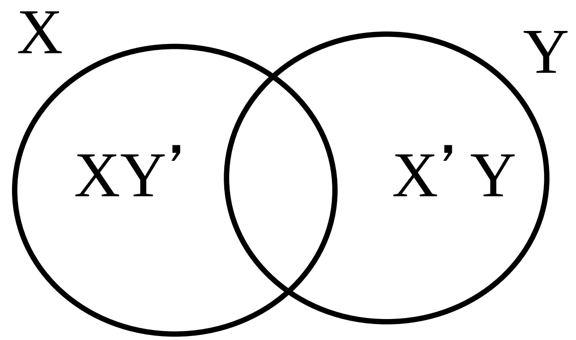


id	x	y	$x \oplus y$
0	0	0	0
1	0	1	1
2	1	0	1
3	1	1	0

	x=0	x=1
y=0	0	1
y=1	1	0

Other Types of Gates: XOR

1) XOR $X \oplus Y = XY' + X'Y$

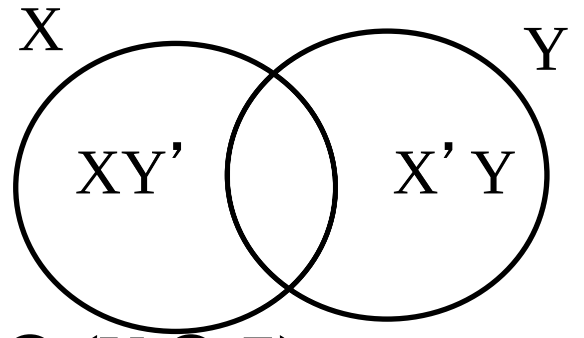


(a) Commutative $X \oplus Y = Y \oplus X$

(b) Associative $(X \oplus Y) \oplus Z = X \oplus (Y \oplus Z)$

Other Types of Gates: XOR

1) XOR $X \oplus Y = XY' + X'Y$



(a) Commutative $X \oplus Y = Y \oplus X$

(b) Associative $(X \oplus Y) \oplus Z = X \oplus (Y \oplus Z)$

(c) $1 \oplus X = X'$, $0 \oplus X = X$

(d) $X \oplus X = 0$, $X \oplus X' = 1$

Other Types of Gates: Properties and Usage

e) If $ab = 0$, then $a \oplus b = a + b$

Proof: If $ab = 0$, then

$a = a(b+b') = ab+ab' = ab'$

$b = b(a + a') = ba + ba' = a'b$

Therefore, $a+b = ab' + a'b = a \oplus b$

In full adder, we have $c_{out}=ab+bc+ac=ab+c(a+b)$

From property e), we can also write $c_{out}=ab+c(a \oplus b)$

Other Types of Gates: XOR

f) $f(x,y) = x \oplus xy' \oplus x'y \oplus (x + y) \oplus x = ?$

(Priority of operations: AND, $\oplus$, OR)

Hint: We apply Shannon's Expansion.

Shannon's Expansion (for switching functions)

Formula: f (X,Y) = X * f (1, Y) + X' * f (0, Y)

Proof by enumeration:

If X = 1, f (X,Y) = f (1, Y) : 1*f (1, Y) + 1' *f(0,Y) = f (1, Y)

If X = 0, f(X,Y) = f (0, Y) : 0*f (1, Y) + 0' *f(0,Y) = f(0, Y)

Other types of gates: XOR

Simplify the function (Priority of operations: AND, $\oplus$, OR)

$f(X,Y) = X \oplus XY' \oplus X'Y \oplus (X+Y) \oplus X$

Case X = 1: $f(1, Y) = 1 \oplus Y' \oplus 0 \oplus 1 \oplus 1 = Y$

Case X = 0: $f(0, Y) = 0 \oplus 0 \oplus Y \oplus Y \oplus 0 = 0$

Thus, using Shannon's expansion, we have

$f(X, Y) = Xf(1,Y)+X'f(0,Y)= XY$

XOR gates

iClicker: Is the equation

$$a+(b\oplus c) = (a+b)\oplus(a+c) \text{ true?}$$

A. Yes

B. No

Other Types of Gates: NAND, NOR

2) NAND, NOR gates

NAND (NOR) gates are not associative

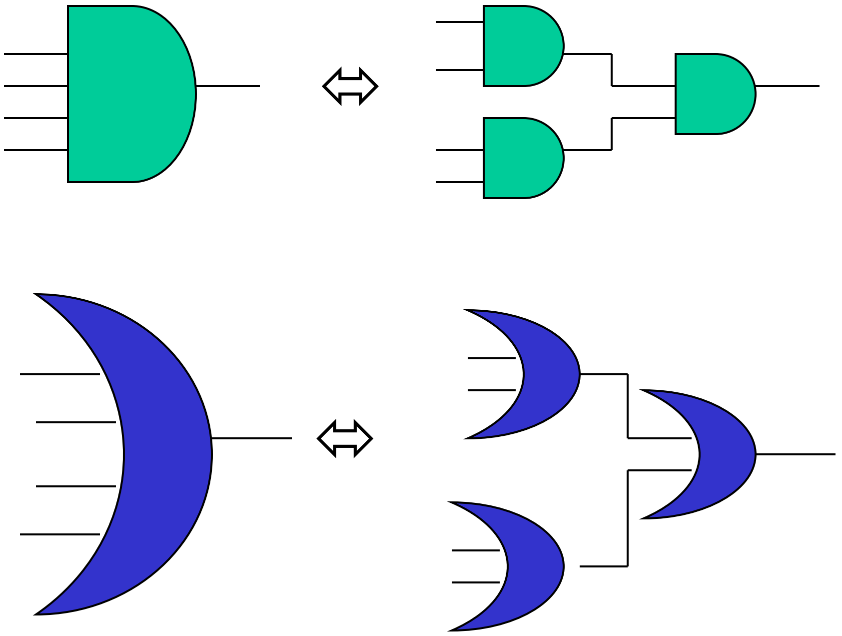
Let $a \mid b = (ab)'$

$$(a \mid b) \mid c \neq a \mid (b \mid c)$$

Other Types of Gates: Block Diagram Transform

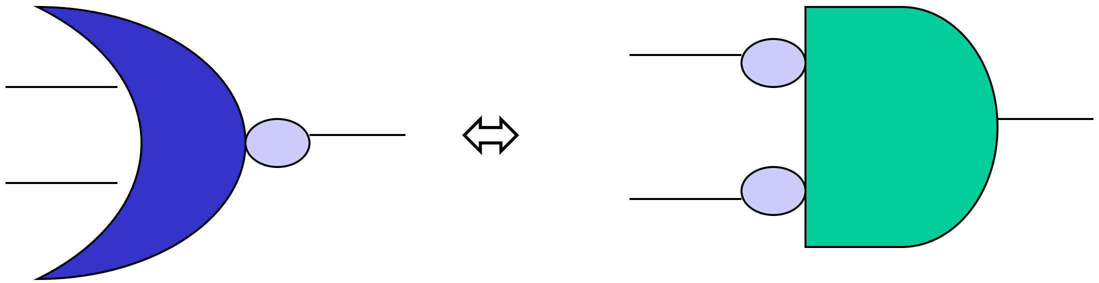
3) Block Diagram Transformation

a) Reduce # of inputs.

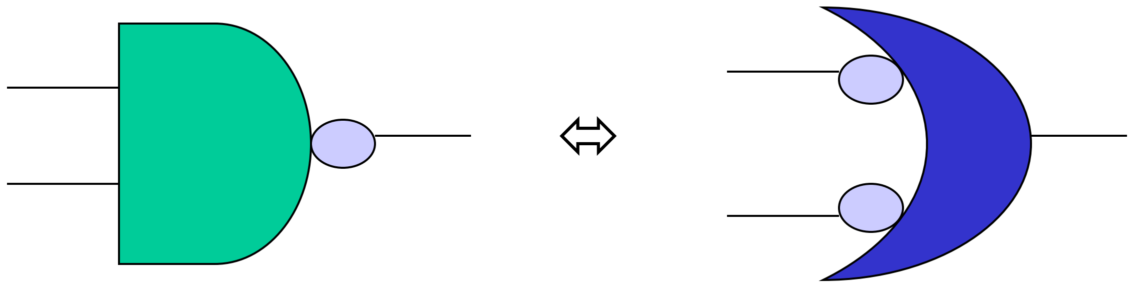


Other Types of Gates: Block Diagram Transform

b. DeMorgan’s Law



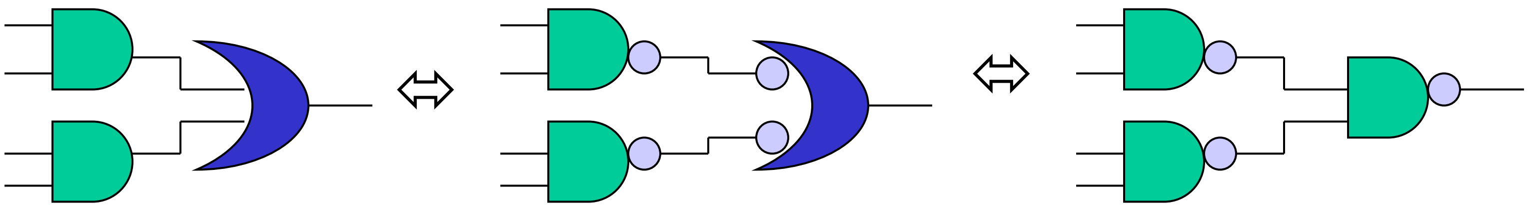
$(a+b)' = a'b'$



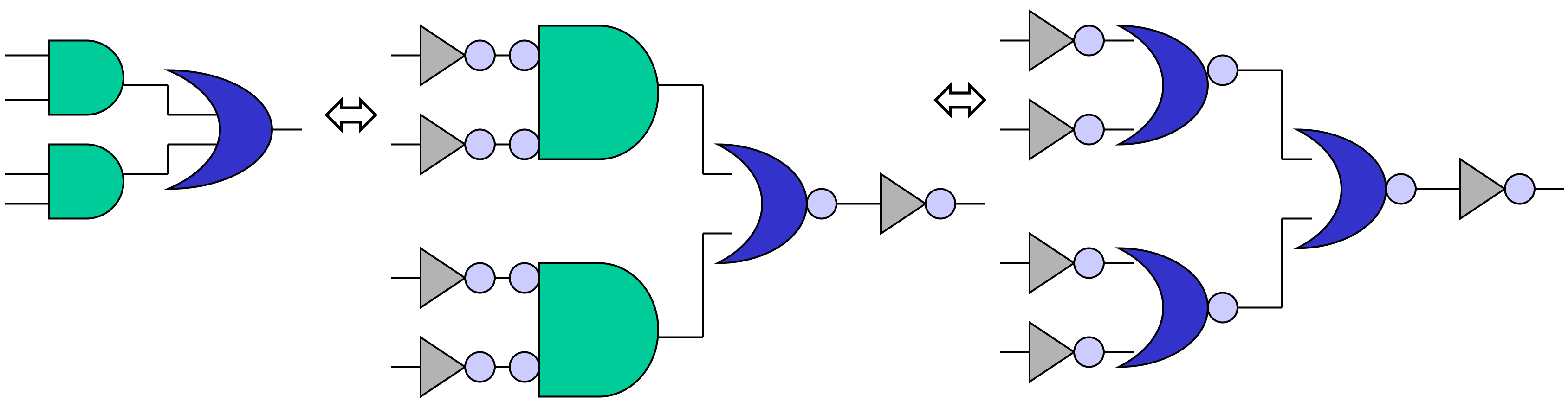
$(ab)' = a' + b'$

Other Types of Gates: Block Diagram Transform

c. Sum of Products (Using only NAND gates)

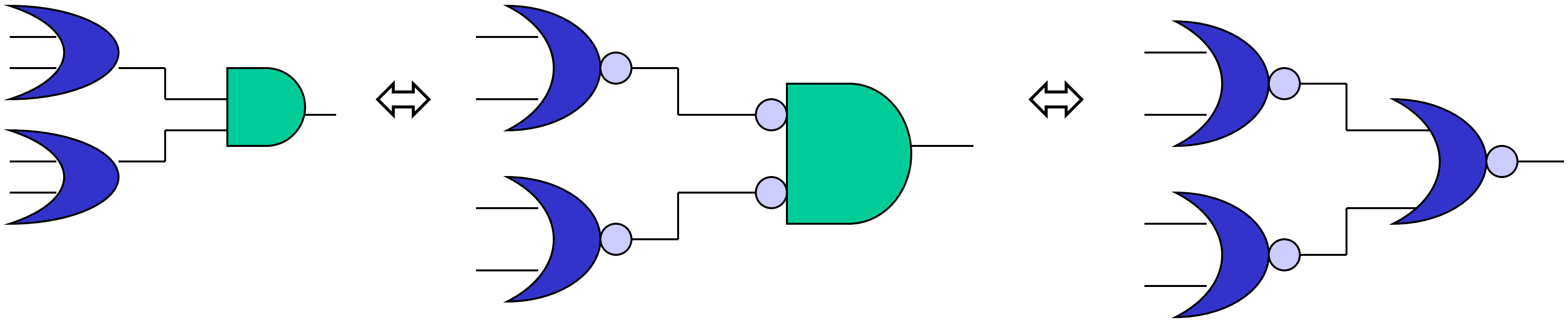


Sum of Products (We create many bubbles with NOR gates)



Other Types of Gates: Block Diagram Transform

d. Product of Sums (NOR gates only)



We will create many bubbles with NAND gates.

Other Types of Gates: Block Diagram Transform

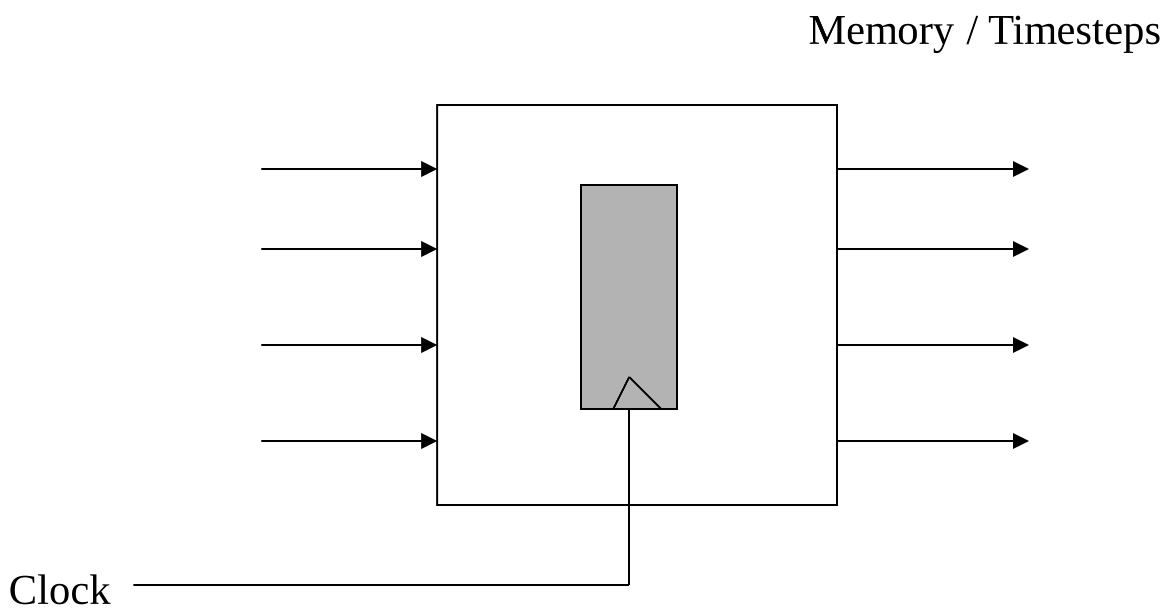
NAND, NOR gates

Remark:

Two level NAND gates: Sum of Products

Two level NOR gates: Product of Sums

Part II. Sequential Networks



Flip flops
Specification
Implementation