1. (Sipser 1.4c) Consider the language

\[ \{w \mid w \text{ has an even number of } a\text{s} \text{ and one or two } b\text{s}\} \].

Construct a DFA recognizing this language. Briefly justify why each state is included and why you chose to make each state either accepting or non-accepting.

*Hint: can you express it as the intersection of two simpler languages?*

*Note: can you design a DFA recognizing this language with fewer states?*
2. **True or False?** Prove your answer.

(a) The set of odd integers is closed under multiplication.

(b) The set of real numbers is closed under taking square root.

(c) The class of regular languages over the alphabet \{0,1\} is closed under the operation $FlipBits(L)$, defined as

$$FlipBits(L) = \{w \mid w \text{ is obtained from some } w' \text{ in } L \text{ by flipping each 0 in } w \text{ to 1, and each 1 to 0}\}$$