1. Let $R$ be a relation with attributes $ABCD$. Consider the CQ algebra query

$$\pi_{AC}[\pi_{AB}(R) \bowtie \pi_{BC}(R)] \bowtie \pi_{CD}(R).$$

(a) (1 point) Convert the above query to a CQ query in rule form.

(b) (3 points) Using the chase, minimize the query obtained in (a) knowing that it is only applied to databases satisfying the FDs

$$A \rightarrow D, \ CD \rightarrow B, \ C \rightarrow A.$$  

(c) (1 point) Construct an algebra query corresponding to the minimized CQ query obtained in (b).

2. (5 points) Let $R$ be a relation over $ABCDE$. Reduce the number of joins in the query

$$\pi_{ACE}(R) \bowtie \pi_{ADE}(R) \bowtie \pi_{BCD}(R)$$

knowing that it is only applied to relations $R$ satisfying the set of constraints

$$\{A \rightarrow B, E \rightarrow D, D \rightarrow E\}.$$  

Proceed as in Problem 1: construct the rule corresponding to the query, chase and minimize its body, and then convert the result back to the algebra.

3. (2 points) Prove or disprove the following statement: it is possible for a set of MVDs to imply a non-trivial FD. More precisely, there exists a set $\Delta$ of MVDs and a non-trivial FD $f$ such that $\Delta \models f$. (An FD is trivial if it is of the form $X \rightarrow Y$ where $Y \subseteq X$; such an FD is always true.) Provide a direct proof, without using the soundness and completeness of the axioms for FDs and MVDs.
4. (3 points) Prove the following inference rule ($R$ has attributes $ABC$ and $S$ has attributes $DEF$):

If $R[AB] \subseteq S[DE], R[AC] \subseteq S[DF]$ and $S$ satisfies $D \rightarrow E$
then $R[ABC] \subseteq S[DEF]$

5. (10 points) Let $CQ^=$ denote the set of conjunctive queries with equality (i.e. CQs allowing atoms $x = c$ and $x = y$ where $x, y$ are variables and $c$ is a constant). Let $\sigma$ be a relational schema. We define the following dependency on instances over $\sigma$, called query inclusion dependency. For $q_1, q_2 \in CQ^=$ over schema $\sigma$, an instance $I$ over $\sigma$ satisfies $q_1 \subseteq q_2$ iff $q_1(I) \subseteq q_2(I)$. The implication problem for $CQ^=$ query inclusions is to determine, given a finite set $\Sigma$ of $CQ^=$ query inclusion dependencies and an additional $CQ^=$ query inclusion dependency $q_1 \subseteq q_2$ over the same schema $\sigma$, whether $\Sigma \models q_1 \subseteq q_2$, meaning that every instance over $\sigma$ satisfying all inclusions in $\Sigma$ also satisfies $q_1 \subseteq q_2$.

(i) (4 points) Show that for every relation schema $R$ and FD $f$ over $R$, there exist $CQ^=$ queries $q, q'$ such that for every instance $I$ over $R$, $I \models f$ iff $q(I) \subseteq q'(I)$.

(ii) (2 points) Show that (i) does not hold if equality is not allowed.

(iii) (4 points) Show that the implication problem for $CQ^=$ query inclusion dependencies is undecidable.

(iv) (brownie point) Do you think the implication problem for $CQ$ query inclusion dependencies (without equality) is decidable? (stream of consciousness intuition is ok here, as long as you don’t say something terribly wrong).