Photometric Stereo

Introduction to Computer Vision
CSE 152
Lecture 4

Announcements

• A larger classroom is not available
  – Enrollment will increase to maximum size
• A tutor will be available
  – Homework assignments will be completed individually
• Homework 0 is due today by 11:59 PM
• Homework 1 will be assigned today
  – Due Wed, Apr 20, 11:59 PM
• Reading:
  – Section 2.2.4: Photometric Stereo
  • Shape from Multiple Shaded Images

Two shape-from-X methods that use shading

• Shape-from-shading: Use just one image to recover shape. Requires knowledge of light source direction and BRDF everywhere. Too restrictive to be useful.

• Photometric stereo: Single viewpoint, multiple images under different lighting.

An example of photometric stereo

surface (albedo textured mapped on surface)

albedo (surface normals)
Photometric stereo

- Single viewpoint, multiple images under different lighting.
  1. Arbitrary known BRDF, known lighting
  2. Lambertian BRDF, known lighting
  3. Lambertian BRDF, unknown lighting.

I. Photometric Stereo: General BRDF and Reflectance Map

BRDF

- Bi-directional Reflectance Distribution Function
  \[ \rho(\theta_{\text{in}}, \phi_{\text{in}}; \theta_{\text{out}}, \phi_{\text{out}}) \]
- Function of
  - Incoming light direction: \(\theta_{\text{in}}, \phi_{\text{in}}\)
  - Outgoing light direction: \(\theta_{\text{out}}, \phi_{\text{out}}\)
- Ratio of incident irradiance to emitted radiance

Coordinate system

Surface: \(\mathbf{s}(x,y) = (x,y,f(x,y))\)

Tangent vectors:
\[ \frac{\partial f}{\partial x}, \quad \frac{\partial f}{\partial y} \]

Normal vector:
\[ \mathbf{n} = \left\{ \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\} \]

Gradient Space (p,q)

Gradient Space: \((p,q)\)
\[ p = \frac{\partial f}{\partial x}, \quad q = \frac{\partial f}{\partial y} \]

Image Formation

For a given point A on the surface, the image irradiance \(E(x,y)\) is a function of

1. The BRDF at \(A\)
2. The surface normal at \(A\)
3. The direction of the light source
Let the BRDF be the same at all points on the surface, and let the light direction \( s \) be a constant.
1. Then image irradiance is a function of only the direction of the surface normal.
2. In gradient space, we have \( E(p, q) \).

Example Reflectance Map: Lambertian surface

For lighting from front

What does the intensity (Irradiance) of one pixel in one image tell us?

It constrains the surface normal projecting to that point to a curve

Three Source Photometric stereo:

**Step 1**
- **Offline:**
  Using source directions & BRDF, construct reflectance map for each light source direction. \( R_1(p, q), R_2(p, q), R_3(p, q) \)
- **Online:**
  1. Acquire three images with known light source directions. \( E_1(x, y), E_2(x, y), E_3(x, y) \)
  2. For each pixel location \( (x, y) \), find \( (p, q) \) as the intersection of the three curves \( R_1(p, q) = E_1(x, y) \), \( R_2(p, q) = E_2(x, y) \), \( R_3(p, q) = E_3(x, y) \)
  3. This is the surface normal at pixel \((x, y)\). Over image, the normal field is estimated
Normal Field

Plastic Baby Doll: Normal Field

Next step:
Go from normal field to surface

Recovering the surface $f(x,y)$

Many methods: Simplest approach
1. From estimate $n=(n_x,n_y,n_z)$, $p=n_x/n_z$, $q=n_y/n_z$
2. Integrate $p=\frac{df}{dx}$ along a row $(x,0)$ to get $f(x,0)$
3. Then integrate $q=\frac{df}{dy}$ along each column starting with value of the first row

II. Photometric Stereo:
Lambertian Surface, Known Lighting

At image location $(u,v)$, the intensity of a pixel $x(u,v)$ is:

$$ e(u,v) = \vec{a}(u,v) \cdot \hat{\vec{n}(u,v)} \cdot s $$

where
- $\vec{a}(u,v)$ is the albedo of the surface projecting to $(u,v)$.
- $\hat{\vec{n}(u,v)}$ is the direction of the surface normal.
- $s$ is the light source intensity.
- $\hat{\vec{s}}$ is the direction to the light source.
Lambertian Photometric stereo

- If the light sources $s_1$, $s_2$, and $s_3$ are known, then we can recover $b$ from as few as three images. (Photometric Stereo: Silver 80, Woodham81).

\[
\begin{bmatrix}
e_1 \\ e_2 \\ e_3
\end{bmatrix} = b^T \begin{bmatrix}s_1 \\ s_2 \\ s_3
\end{bmatrix}
\]

- i.e., we measure $e_1$, $e_2$, and $e_3$ and we know $s_1$, $s_2$, and $s_3$. We can then solve for $b$ by solving a linear system. $b^T = \begin{bmatrix}e_1 & e_2 & e_3 \end{bmatrix}$

- Normal $\hat{n} = b / |b|$ and albedo $a = |b|

What if we have more than 3 Images?

Linear Least Squares

\[
\begin{bmatrix}e_1 & e_2 & \ldots & e_n\end{bmatrix} = b^T \begin{bmatrix}s_1 & s_2 & \ldots & s_n\end{bmatrix}
\]

Let the residual be $r = e - Sb$

Squaring this:

\[
\begin{align*}
0 &= r^T r = (e - Sb)^T (e - Sb) \\
&= e^T e - 2b^T St e + b^T S St b
\end{align*}
\]

where $e$ is $n$ by 1, $b$ is 3 by 1, $S$ is $n$ by 3

Solving for $b$ gives

\[
b = (S^T S)^{-1} S^T e
\]
III. Photometric Stereo with unknown lighting and Lambertian surfaces

How do you construct subspace?

\[
\begin{bmatrix}
E_1 & E_2 & E_3
\end{bmatrix} = B^T \begin{bmatrix}
s_1 & s_2 & s_3
\end{bmatrix}
\]

- Given three or more images \(E_1 \ldots E_n\), estimate \(B\) and \(s_i\).
- How? Given images in form of \(E = [E_1 \ldots E_n]\), Compute \([U, S, V] = \text{SVD}(E)\) and \(B^*\) is the \(n\) by 3 matrix formed by first 3 columns of \(U\).

Do Ambiguities Exist? Yes

- Is \(B\) unique?
- For any invertible matrix \(A\), \(B^* = BA\) also a solution
- For any image of \(B\) produced with light source \(S\), the same image can be produced by lighting \(B^* = BA\) with \(S^* = A^{-1}S\) because \(X = B^*S^* = BAA^{-1}S = BS\)
- When we estimate \(B\) using Singular Value Decomposition (SVD), the rows are NOT generally the normal times the albedo.
GBR Transformation

Only Generalized Bas-Relief transformations satisfy the integrability constraint:

\[
A = G^T = \begin{bmatrix} 1 & 0 & -\mu \\ 0 & 1 & -\nu \\ 0 & 0 & 1 \end{bmatrix}
\]

\[
f(x,y) = \hat{f}(x,y) + \mu \alpha + \nu \beta
\]

Generalized Bas-Relief Transformations

Without knowledge of light source location, one can only recover surfaces up to GBR transformations.

\[
f(x,y) = \hat{f}(x,y) + \mu \alpha + \nu \beta
\]

f: true depth
\(\hat{f}\): GBR transform of depth

Uncalibrated photometric stereo

1. Take n images as input without knowledge of light directions or strengths
2. Perform SVD to compute \(B^*\).
3. Find some A such that \(B^*A\) is close to integrable.
4. Integrate resulting gradient field to obtain height function \(f^*(x,y)\).

Comments:
- \(f^*(x,y)\) differs from \(f(x,y)\) by a GBR.
- Can use specularities to resolve GBR for non-Lambertian surface.

Next Lecture

- Binary image processing
- Reading:
  - Szeliski, Chapter 3 Image processing, Section 3.3 More neighborhood operators