CSE 202: Algorithm Design and Analysis Spring 2015

Problem Set 3

Instructor: Fan Chung Graham **Due on:** Wed, May 20, 2015

Instructions

- For your proofs, you may use any lower bound, algorithm or data structure from the text or in class, and their correctness and analysis, but please cite the result that you use.
- If you do not prove that your algorithm is correct, we will assume that it is incorrect. If you do not provide an analysis of the running time, we will assume you do not know what the running time is.

Problem 1

Let M be a $n \times n$ matrix in which each entry is equal to either 0 or 1. Let m_{ij} denote the entry of M at row i and column j.

For $1 \leq i, j \leq n$, swapping rows i and j denotes the following action: we swap the values of $m_{i,k}$ and $m_{j,k}$ for all $k = 1, \ldots, n$. Swapping columns i and j are also defined analogously. We say that M is *rearrangable* if it is possible to swap some pairs of rows and some pairs of columns in any sequence such that after swapping, all the diagonal entries m_{ii} are equal to 1.

1. Give an example of a matrix M which is not rearrangeable, yet for which at least one entry in each row and column is equal to 1.
2. Design an algorithm that determines whether a matrix M with $0-1$ entries is rearrangeable. (Hint: Try to set it up as a bipartite matching problem.)

Problem 2

In a public building such as a movie theater, it is important to have a plan of exit in the event of a fire. We will design such an emergency exit plan in this question using max-flows. Suppose a movie theater is represented by a graph $G = (V, E)$, where each room, landing, or other location is represented by a vertex and each corridor or stairway is represented by an edge. Each corridor has an associated capacity c, meaning that at most c people can pass through the corridor at once. Traversing a corridor from one end to the other takes one timestep. (Traversing a room takes zero time.)

1. Suppose all people are initially in a single room s, and there is a single exit t. Show how to use maximum flow to find a fastest way to get everyone out of the building. (Hint: create another graph G' that has vertices to represent each room at each time step.)
2. Show how the same idea can be used when people are initially in multiple locations and there are multiple exits.
3. Finally, suppose that it takes different (but integer) amounts of time to cross different corridors or stairways, and that for each such corridor or stairway e, you are also given an integer $t(e)$ which is the number of seconds required to cross e. Now show how to transform your algorithm in Part (1) to find a fastest way to get everyone out of the building.