Image Formation and Cameras

Introduction to Computer Vision
CSE 152
Lecture 4

Announcements
http://cseweb.ucsd.edu/classes/sp15/cse152-a/
• Piazza
• Homework 1 will be assigned today
• Wait list, additional TA/Tutor, larger room
• Reading:
  – Chapter 2 Image formation

Image Formation: Outline
• Factors in producing images
• Projection
• Perspective
• Vanishing points
• Orthographic
• Lenses
• Sensors
• Quantization/Resolution
• Illumination
• Reflectance

Earliest Surviving Photograph
• First photograph on record, “la table service” by Nicephore Niepce in 1822.
• Note: First photograph by Niepce was in 1816.

How Cameras Produce Images
• Basic process:
  – photons hit a detector
  – the detector becomes charged
  – the charge is read out as brightness
• Sensor types:
  – CCD (charge-coupled device)
    • high sensitivity
    • high power
    • cannot be individually addressed
    • blooming
  – CMOS
    • simple to fabricate (cheap)
    • lower sensitivity, lower power
    • can be individually addressed

Images are two-dimensional patterns of brightness values.
Pinhole Camera: Perspective projection

- Abstract camera model - box with a small hole in it

"When images of illuminated objects ... penetrate through a small hole into a very dark room ... you will see [on the opposite wall] these objects in their proper form and color, reduced in size ... in a reversed position, owing to the intersection of the rays". --- Leonardo Da Vinci

Camera Obscura

- Used to observe eclipses (e.g., Bacon, 1214-1294)
- By artists (e.g., Vermeer).

Camera Obscura

Jetty at Margate England, 1898.
Distant objects are smaller

Geometric properties of projection
- 3-D points map to points
- 3-D lines map to lines
- Planes map to whole image or half-plane
- Polygons map to polygons

Important point to note: Angles & distances not preserved, nor are inequalities of angles & distances.
Degenerate cases:
- line through focal point projects to point
- plane through focal point projects to a line

In the perspective image, two parallel lines meet at a point

Parallel lines meet in the image
- Formed by line through O
- Parallel to the given line(s)
- A single line can have a vanishing point

Projective geometry provides an elegant means for handling these different situations in a unified way, and homogenous coordinates are a way to represent entities (points & lines) in projective spaces.
Vanishing Points

Equation of Perspective Projection

Cartesian coordinates:
• We have, by similar triangles, that \((x, y, z) \rightarrow (f x/z, f y/z, \ -f)\)
• Ignoring the third coordinate, we get \((x, y, z) \rightarrow (f x/z, f y/z)\)

The equation of projection

Cartesian coordinates:
\[
\begin{bmatrix}
U \\
V \\
W
\end{bmatrix} = \begin{bmatrix}
10 & 0 & 0 \\
0 & 10 & 0 \\
0 & 0 & 10
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}
\]

Equation of Perspective Projection

Cartesian coordinates:
• We have, by similar triangles, that \((x', y', z') = (f' x/z, f' y/z, f')\)
• Establishing an image plane coordinate system at \(C'\) aligned with \(i\) and \(j\), image coordinates of the projection of \(P\) are \((x', y', z') \rightarrow (f' x/z, f' y/z)\)

Simplified Camera Models

Perspective Projection

Affine Camera Model

• Take perspective projection equation, and perform Taylor series expansion about some point \(P = (x_0, y_0, z_0)\).
• Drop terms that are higher order than linear.
• Resulting expression is called the affine camera model.

Appropriate in Neighborhood About \((x_0, y_0, z_0)\)
(xo, y0, z0) = (0, 0, z0) – a point on the optical axis

Starting with Affine Camera Model, take Taylor series about (x0, y0, z0)

\[
\begin{bmatrix}
  u \\
v
\end{bmatrix} = f \begin{bmatrix}
x \\
y \\
z
\end{bmatrix} + \frac{f}{z_0} \begin{bmatrix}
x - x_0 \\
y - y_0 \\
z - z_0
\end{bmatrix} + \frac{1}{2} \begin{bmatrix}
 0 & 0 & -f_x/z_0^2 \\
0 & 0 & -f_y/z_0^2 \\
-1 & 0 & z_0/z_0^2
\end{bmatrix} \begin{bmatrix}
x - x_0 \\
y - y_0 \\
z - z_0
\end{bmatrix} + \ldots
\]

• Dropping higher order terms and regrouping.

\[
\begin{bmatrix}
  u \\
v
\end{bmatrix} = f \begin{bmatrix}
x \\
y \\
z
\end{bmatrix} + \frac{f}{z_0} \begin{bmatrix}
x - x_0 \\
y - y_0 \\
z - z_0
\end{bmatrix} + \begin{bmatrix}
f / z_0 & 0 & -f_x/z_0^2 \\
0 & f / z_0 & -f_y/z_0^2 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
x - x_0 \\
y - y_0 \\
z - z_0
\end{bmatrix} + \ldots
\]

– That is the x coordinate is dropped, and the image a scaling of the x and y coordinates, where the scale is 1/z0 the depth of the point of the expansion.

The projection matrix for scaled orthographic projection

\[
\begin{bmatrix}
  U \\
  V \\
  W
\end{bmatrix} = \begin{bmatrix}
f / z_0 & 0 & 0 & 0 \\
0 & f / z_0 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
\]

• Parallel lines project to parallel lines
• Ratios of distances are preserved under orthographic projection

For all cameras?

Other camera models

• Generalized camera – maps points lying on rays and maps them to points on the image plane.

Omnican (hemispherical) Light Probe (spherical)
Some Alternative “Cameras”