Photometric Stereo
Introduction to Computer Vision
CSE 152
Lecture 15

Announcements
• Final exam will be a take home exam
• Homework 2 has been graded
• Homework 3 is due May 29, 11:59 PM
• Homework 4 will be assigned this week

• Reading:
  – Section 12.1.1 Shape from shading and photometric stereo

Two shape-from-X methods that use shading
• Shape-from-shading: Use just one image to recover shape. Requires knowledge of light source direction and BRDF everywhere. Too restrictive to be useful.

• Photometric stereo: Single viewpoint, multiple images under different lighting.
An example of photometric stereo

Multi-view stereo vs. Photometric Stereo:
Assumptions
• Multi-view Stereo
  – Multiple images
  – Dynamic scene
  – Multiple viewpoints
  – Fixed lighting
• Photometric Stereo
  – Multiple images
  – Static scene
  – Fixed viewpoint
  – Multiple lighting conditions

Photometric stereo

Photometric stereo
• Single viewpoint, multiple images under
different lighting.
  1. Arbitrary known BRDF, known lighting
  2. Lambertian BRDF, known lighting
  3. Lambertian BRDF, unknown lighting.

I. Photometric Stereo:
General BRDF
and Reflectance Map

BRDF
• Bi-directional Reflectance
  Distribution Function
  \( \rho(\theta_{in}, \phi_{in}; \theta_{out}, \phi_{out}) \)
• Function of
  – Incoming light direction:
    \( \theta_{in}, \phi_{in} \)
  – Outgoing light direction:
    \( \theta_{out}, \phi_{out} \)
• Ratio of incident irradiance to
  emitted radiance

Coordinate system
Surface: \( s(x,y) = (x,y, f(x,y)) \)
Tangent vectors:
\[
\frac{\partial s(x,y)}{\partial x} = \left( \begin{array}{c} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{array} \right)
\]
\[
\frac{\partial s(x,y)}{\partial y} = \left( \begin{array}{c} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{array} \right)
\]
Normal vector
\[
\mathbf{n} = \frac{\mathbf{r}_x \times \mathbf{r}_y}{\| \mathbf{r}_x \times \mathbf{r}_y \|}
\]
Gradient Space (p,q)

\[ p = \frac{\partial f}{\partial x}, \quad q = \frac{\partial f}{\partial y} \]

Normal vector

\[ \hat{n} = \frac{1}{\sqrt{p^2 + q^2}} \begin{pmatrix} p \\ q \\ 1 \end{pmatrix} \]

Image Formation

For a given point A on the surface, the image irradiance \( E(x,y) \) is a function of

1. The BRDF at A
2. The surface normal at A
3. The direction of the light source

Reflectance Map

Let the BRDF be the same at all points on the surface, and let the light direction \( s \) be a constant.

1. Then image irradiance is a function of only the direction of the surface normal.
2. In gradient space, we have \( E(p,q) \).

Example Reflectance Map: Lambertian surface

For lighting from front

Lambertian Reflectance Map

\[ E(p,q) = \frac{1}{p^2 + q^2} \]

Reflectance Map of Lambertian Surface

What does the intensity (Irradiance) of one pixel in one image tell us?

* It constrains the surface normal projecting to that point to a curve.
Two Light Sources

Two reflectance maps

A third image would disambiguate match

Three Source Photometric stereo:

Step 1

Offline:
Using source directions & BRDF, construct reflectance map for each light source direction. \( R_1(p,q), R_2(p,q), R_3(p,q) \)

Online:
1. Acquire three images with known light source directions. \( E_1(x,y), E_2(x,y), E_3(x,y) \)
2. For each pixel location \((x,y)\), find \((p,q)\) as the intersection of the three curves
   \[ R_1(p,q) = E_1(x,y) \]
   \[ R_2(p,q) = E_2(x,y) \]
   \[ R_3(p,q) = E_3(x,y) \]
3. This is the surface normal at pixel \((x,y)\). Over image, the normal field is estimated

Normal Field

Plastic Baby Doll: Normal Field

Next step:
Go from normal field to surface

Recovering the surface \( f(x,y) \)

Many methods: Simplest approach
1. From estimate \( \mathbf{n} = (n_x, n_y, n_z) \), \( p = n_x/n_z, q = n_y/n_z \)
2. Integrate \( p = df/dx \) along a row \((x,0)\) to get \( f(x,0) \)
3. Then integrate \( q = df/dy \) along each column starting with value of the first row

\[ f(x,0) \]
II. Photometric Stereo:
Lambertian Surface, Known Lighting

At image location \( (u,v) \), the intensity of a pixel \( x(u,v) \) is:

\[
e(u,v) = [a(u,v) \hat{n}(u,v)] \cdot [s_0 s]
\]

where

- \( a(u,v) \) is the albedo of the surface projecting to \( (u,v) \).
- \( \hat{n}(u,v) \) is the direction of the surface normal.
- \( s_0 \) is the light source intensity.
- \( \hat{s} \) is the direction to the light source.

If the light sources \( s_1, s_2, \) and \( s_3 \) are known, then we can recover \( b \) from as few as three images. (Photometric Stereo: Silver 80, Woodham 81).

\[
[e_1 e_2 e_3] = b^T [s_1 s_2 s_3] \]

i.e., we measure \( e_1, e_2, \) and \( e_3 \) and we know \( s_1, s_2, \) and \( s_3 \). We can then solve for \( b \) by solving a linear system.

\[
b^T = [e_1 e_2 e_3] [s_1 s_2 s_3]^{-1}
\]

Normal is: \( n = b/|b| \), albedo is: \( |b| \)

What if we have more than 3 Images?
Linear Least Squares

Let the residual be

\[
r = e - Sb
\]

Squaring this:

\[
r^2 = (e - Sb)^T (e - Sb)
\]

Rewrite as

\[
e = Sb
\]

where

- \( e \) is \( n \) by 1
- \( b \) is 3 by 1
- \( S \) is \( n \) by 3

Solving for \( b \) gives

\[
b = (S^T S)^{-1} S^T e
\]
III. Photometric Stereo with unknown lighting and Lambertian surfaces

How do you construct subspace?

\[
\begin{bmatrix}
E_1 & E_2 & E_3 \\
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
= B^T [s_1 \ s_2 \ s_3]
\]

- Given three or more images $E_1 \ldots E_n$, estimate $B$ and $s_i$.
- How? Given images in form of $E = [E_1, E_2 \ldots]$, Compute $[U,S,V] = \text{SVD}(E)$ and $B^*$ is the $n$ by 3 matrix formed by first 3 columns of $U$. 

Face Basis

Original Images

Basis Images
**Render Image from Basis Images**

- Rendered Image (Single Light Source)

**Do Ambiguities Exist? Yes**

- Is B unique?
- For any invertible matrix $A$, $B^* = BA$ also a solution
- For any image of $B$ produced with light source $S$, the same image can be produced by lighting $B^* = BA$ with $S^* = A^{-1}S$ because $X = B^*S^* = BAA^{-1}S = BS$
- When we estimate B using Singular Value Decomposition (SVD), the rows are NOT generally the normal times the albedo.

**GBR Transformation**

Only **Generalized Bas-Relief** transformations satisfy the integrability constraint:

$$A = G^T = \begin{bmatrix} \lambda & 0 & -\mu \\ 0 & \lambda & -\nu \\ 0 & 0 & 1 \end{bmatrix}$$

**Generalized Bas-Relief Transformations**

Without knowledge of light source location, one can only recover surfaces up to GBR transformations.

$$f(x, y) = f'(x, y) + \mu x + \nu y$$

$f$: true depth
$f'$: GBR transform of depth

**Uncalibrated photometric stereo**

1. Take $n$ images as input without knowledge of light directions or strengths
2. Perform SVD to compute $B^*$.
3. Find some $A$ such that $B^*A$ is close to integrable.
4. Integrate resulting gradient field to obtain height function $f^*(x, y)$.

Comments:
- $f^*(x, y)$ differs from $f(x, y)$ by a GBR.
- Can use specularities to resolve GBR for non-Lambertian surface.