CSE 140 Midterm 1 version B
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Spring 2015

Name of the person on your left: ________________________________

Name of the person on your right: __________________________________

- Do not start the exam until you are told.
- Write your name and PID at the top of every page. Write the names of people on your left and right on the first page.
- Turn off and put away all your electronics. This is a closed-book, closed-notes. You may only refer to one 8 ½ x 11” page of your handwritten notes.
- By turning in this exam for grading you are stating that you have followed the UCSD’s academic honesty policies. Do not look at anyone else’s exam or talk to anyone but an exam proctor.
- If you have a question, raise your hand and an exam proctor will come to you.
- You have 80 minutes to finish the exam. When the time is finished, you must stop writing.
- Write your answers in the space provided. To get the most partial credit, clearly show all steps of your work.

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Problem 1
Universal Gates

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a) Given the above K-Map, derive the minimal Boolean expression in SOP.

\[ a'c + bc \]

5 points

b) Implement the same expression using only NAND gates and inverters.

![NAND gates and inverters diagram]

7 points

c) How many transistors does the NAND/INV implementation require?

16 transistors (or 14 transistors)

3 points
Problem 2
K-Maps

Perform two-level logic size optimization for $F(a,b,c,d) = a'b'c'd' + a'bc'd' + abc'd' + abc'd + ab'c'd$ using a Kmap, assuming $c$ and $d$ can never be both 1.

a. List all the prime implicants

The Kmap of $F(a,b,c,d) = a'b'c'd' + a'bc'd' + abc'd' + abc'd + ab'c'd$ is

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Prime implicants: $a'c'd'$, $bc'd'$, $abc'$, $ad$
10 points maximum (5 for kmap and 5 for prime implicants)

b. List all the essential primes

Essential implicants: $a'c'd'$, $ad$
3 points (-1.5 for each mistake)

c. Show the minimal cover in Boolean equation format.

Minimal cover: $a'c'd'+ad+bc'd'$ OR $a'c'd'+ad+abc'$
2 points
Problem 3  
Boolean Logic

A logic network has three inputs (A, B, C) and one output (Z). In the following situations we set the output Z to either a logic zero or one:

1. The output Z is logic 0 when the binary value of ABC is greater than 3 and odd.
2. When the binary value of ABC is greater than 3 and even, Z is a logic 1.
3. When the binary value of ABC is less than 3, the output Z follows the result of the expression $B \oplus C$

a. Fill in the truth table

<table>
<thead>
<tr>
<th>A</th>
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Max 8 points: +1 for each correct entry in truth table (1*8 = 8)

b. Write the canonical SOP of function Z using standard $\Sigma m( ) + DC( )$ notation:

$$Z(A,B,C) = \Sigma m(1,2,4,6) + DC(3)$$

Max 5 points: +1 for each correct minterm (1*5= 5)

c. Write the minimum SOP expression for the function Z:

$$Z(A,B,C) = A'C + BC' + AC' \quad \text{OR} \quad A'C + A'B + AC'$$

Max 2 points : +1 for each correct implicant (1*2 = 2).
Partial credit if some implicants are correct.
**Problem 4**  
Logic minimization

Derive the minimal form of the following function using Boolean algebra (or Shannon’s expansion).

\[ f(A,B,C) = (a + b)(a + c')(b + c) \]

**method 1:**
\[ f(A,B,C) = (a + b)(a + c')(b + c) \]
\[ = (a + ac' + ab + bc')(b + c) \quad \text{(get 3 points)} \]
\[ = ab + ac + abc' + acc' + ab + abc + bc' + bcc' \quad \text{(if all 8 implicates right, get 5 points)} \]
\[ = ab + ac + bc' \quad \text{(eliminate redundant terms and have three left, get 12 points)} \]
\[ = (ab + ac) + (c'b + c'c) \]
\[ = a(b + c) + c'(b + c) \]
\[ = (a + c')(b + c) \quad \text{(get 15 points)} \]

**method 2:**
\[ f(A,B,C) = (a + b)(a + c')(b + c) \]
\[ = (a + ac' + ab + bc')(b + c) \quad \text{(get 3 points)} \]
\[ = ab + ac + abc' + acc' + ab + abc + bc' + bcc' \quad \text{(if all 8 implicates right, get 5 points)} \]
\[ = ab + ac + bc' \quad \text{(eliminate redundant terms and have three left, get 12 points)} \]
\[ = ac + bc' \quad \text{(by consensus theorem, get 15 points)} \]

**method 3:**
Use Shannon's expansion:
\[ f(A,B,C) = (a + b)(a + c')(b + c) \]
\[ = (c + f(a,b,0))(c' + f(a,b,1)) \quad \text{(get 5 points if correct Shannon’s expansion)} \]
\[ = (c + (a + b)b)(c' + (a + b)a) \quad \text{(get 12 points)} \]
\[ = (c + b)(c' + a) \]
\[ = (a + c')(b + c) \quad \text{(get 15 points)} \]

Grading: Total 15 points. If you use kmap instead of Boolean algebra, get 3 points. If there is any mistake after the expansion, get 3 points only.
Problem 5
Circuit Analysis

Z is a function of inputs A, B and C as shown in the circuit below.

a) Fill in the truth table.

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Max 16 points. +2 for each correct entry in the truth table (2*8 = 16)

b) Write the minimum size Boolean equation for Z.

\[
Z(A,B,C,D) = ((A'B'C')' + B)' + B \\
= ((A' + B + C')' + B) \quad \text{De Morgan's Law} \\
= (A' + B + C')' + B \\
= (A'B'C' + B) \quad \text{De Morgan's Law} \\
= A'C + B \quad \text{Minimization}
\]

Max 4 points : +2 for each correct implicant (2*2 = 4)
Problem 6
Multiplexer

For the Boolean function \( f(a,b,c) = \overline{M}(1,2,3) \), find the missing inputs to the multiplexers. Assume EN=1.

Answer:

Grading: Total 20 points. +10 points for each correct answer. +5 points for the truth table or K map if both the answers are wrong.
First Name:        Last Name:        PID:

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