1. Solve the following:
   a. Draw a circuit using AND, OR and NOT gates for the following equation:
      \[ F(a,b,c) = (ab) (b' + c) \]
   b. Convert the circuit using only NAND gates (INV are ok)
   c. Convert the circuit using only NOR gates (INV are ok)

Solution:

**PLEASE NOTE:** solutions including NOT gates (instead of NORs or NANDs connected with both inputs to the same wire) are fine as well.

a. Draw a circuit using AND, OR and NOT gates for the following equation:
   \[ F(a,b,c,d) = (ab) (b' + c) \]

b. Convert the circuit using only NAND gates and INV
c. Convert the circuit using only NOR gates and INV

2. a. Use DeMorgan’s Law to find the inverse of the following equation:
   \[ F = abc + a'b. \]
   Reduce to minimal number of product-terms (SOP form).
   
   Hint: Start with \( F' = (abc + a'b)' \).

b. Use DeMorgan’s Law to find the inverse of the following equation:
   \[ F = a'c+a'b'd+cd'. \]
   Reduce to minimal number of sum-terms (product-of-sums form).

Solution:
   a. \( F = abc + a'b \Rightarrow \)
      \[ F' = (abc + a'b)' = (abc)'(a'b)' = (a'+b'+c')(a+b') = a'b' + ab' + b' + ac' + b'c' = b' + ac' \]
   b. \( F = a'c+a'b'd+cd' \Rightarrow \)
      \[ F' = (a'c+a'b'd+cd')' = (a'c)'(a'b'd')(cd')' = (a+c')(a+b+d')(c'+d) \]

3. Given the following function:
   \[ F = (A'. B'. C')'. C + (A'. B'. C')' + D \]
   Using Boolean Algebra, prove that the logic equation above can be implemented by a 4-input OR gate:
   \[ F = A + B + C + D \]
   Show ALL the steps of your proof AND state the name of the axiom or theorem used in each step. Apply only one axiom/theorem at each step.
Solution:

\[ F = (A'B'C')'C + (A'B'C')' + D \]

\[ = (A'' + B'' + C'')C + (A'' + B'' + C'') + D \] 

DeMorgan's

\[ = (A + B + C)C + (A + B + C) + D \]

Involution

\[ = AC + BC + CC + A + B + C + D \]

Distributivity

\[ = AC + BC + C + A + B + C + D \]

Idempotency

\[ = AC + BC + C + A + B + D \]

Commutativity

\[ = AC + BC + C + A + B + D \]

Idempotency

\[ = A + B + C + D \]

Commutativity

OR

\[ F = (A'B'C')'C + (A'B'C')' + D \]

\[ = (A'' + B'' + C'')C + (A'' + B'' + C'') + D \] 

DeMorgan's

\[ = (A + B + C)C + (A + B + C) + D \]

Involution

\[ = (A + B + C)(C + 1) + D \]

Distributivity

\[ = (A + B + C).1 + D \]

Null Element

\[ = (A.1 + B.1 + C.1) + D \]

Distributivity

\[ = (A + B + C) + D \]

Identity

\[ = A + B + C + D \]

Associativity

4. A museum has three rooms each with a motion sensor (m0, m1, and m2) that outputs 1 when motion is detected. At night, the only person in the museum is one security guard who walks from room to room. Create a function that sounds an alarm (by setting an output A to 1) if motion is detected in more than one room at a time (i.e., in two or three rooms), meaning that there must be one or more intruders in the museum.

a. Specify the truth table

b. formulate canonical sum of products implementation for A

c. Minimize A using K-maps and write the minimum implementation of A

d. Draw the circuit

Solution:
In canonical SOP form, $A = \sum m(3,5,6,7)$

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Minimal form: $A = m_1m_2 + m_0m_1 + m_0m_2$