Quiz 1b Solutions

1 Convert \([31021]_4\) to base 5.

Let us first convert the number in base 4 to base 10.
\([31021]_4 = 841\) in base 10. Let us then convert 841 into base 5.
\[841 = [11331]_5.\]

2 What is the sum of \([24665]_7\) and \([63606]_7\) in the base 7 representation.

\([24665]_7 + [63606]_7 = [121604]_7.\]

3 What is the maximum integer that can be represented in base 4 using only 5 bits (that is, what is the largest integer which when represented in base 4 has at most length 5 representation).

Since the largest digit we can use in base 4 representation is 3, the largest integer we can have using only 5 bits will be \([33333]_4\), which equals 1023 in base 10.

4 If \(a, b, c\) are three positive integers such that \(a\) divides \(b\) then prove that \(ac\) divides \(bc\).

If \(a\) divides \(b\), then \(b = ak, k \in \mathbb{Z}^+\). Let us multiply both sides by \(c\). Then, \(bc = akc = (ac)k, k \in \mathbb{Z}^+\). Since \(bc\) is a multiple of \(ac\), \(ac\) divides \(bc\).

5 Let \(a\) and \(b\) be two positive integers such that 2 is the remainder when both \(a\) and \(b\) are divided by 6. Prove that \(ab + 2\) is divisible by 6.

\(a = 6k + 2, k \in \mathbb{Z}^+\) and \(b = 6m + 2, m \in \mathbb{Z}^+\). Therefore,
\[ab + 2 = (6k + 2)(6m + 2) + 2\]
\[= 36km + 12k + 12m + 6 = 6(6km + 2k + 2m + 1)\]
Since \((6km + 2k + 2m + 1) \in \mathbb{Z}^+, ab + 2\) is also divisible by 6.