1. Let $r$ = “she registered to vote” and $v$ = “she voted”. Write the following statement in symbolic form: She registered to vote but she did not vote.

Solution. $r \land \sim v$

2. Make a truth table for $(p \lor (\sim p \lor q)) \land (q \land \sim r)$

Solution.

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$r$</th>
<th>$\sim p$</th>
<th>$\sim p \lor q$</th>
<th>$p \lor (\sim p \lor q)$</th>
<th>$\sim r$</th>
<th>$q \land \sim r$</th>
<th>$(p \lor (\sim p \lor q)) \land (q \land \sim r)$</th>
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3. Using DeMorgan’s rule, state the negation of the statement: “The car is out of gas or the fuel line is plugged.”

Solution. Let $p$ = “the car is out of gas” and $q$ = “the fuel line is plugged.” Then, the statement $s = p \lor q$. The negation of $s$ is $\sim s = \sim (p \lor q) = \sim p \land \sim q$ by DeMorgan’s rule. So the negation reads “The car is not out of gas and the fuel line is not plugged.”

4. A pair of numbers $x$ and $y$ satisfy a system of inequalities if

\[
\begin{cases}
3 \leq x \leq 5 \\
|x - y| < 1.
\end{cases}
\]

What are the conditions under which $x$ and $y$ fail to satisfy this system?
Solution. This system can also be written as a conjunction of two statements. Let $p$ be the statement $3 \leq x \leq 5$ and $q$ be the statement $|x - y| < 1$. Then $\sim (p \land q) = \sim p \lor \sim q$ by DeMorgan’s rule. So system fails when $x < 3$ or $x > 5$ or $|x - 1| \geq 1$.

5. Is the function $(p \land (\sim (\sim p \lor q))) \lor (p \land q)$ equal to the function $p \lor q$? Why or why not?

Solution. No.

I will use two methods to solve this question. The first way is uses truth tables and the second way uses the algebraic rules.

Method 1

<table>
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<tr>
<th>$p$</th>
<th>$q$</th>
<th>$\sim p$</th>
<th>$\sim (p \lor q)$</th>
<th>$p \land (\sim (p \lor q))$</th>
<th>$p \land q$</th>
<th>$(p \land (\sim (p \lor q))) \lor (p \land q)$</th>
<th>$p \lor q$</th>
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</table>

The last two columns of the table are not the same, and thus, the two statements are not equivalent.

Method 2

$(p \land (\sim (\sim p \lor q))) \lor (p \land q)$

$(p \land (\sim p \land \sim q)) \lor (p \land q)$ DeMorgan’s rule

$(p \land (p \land \sim q)) \lor (p \land q)$ double negation

$((p \land p) \land \sim q) \lor (p \land q)$ associative rule

$(p \land \sim q)) \lor (p \land q)$ idempotent

$p \land (\sim q \lor q)$ distributive rule

$p \land T$ negation

$p$ bound rule

Since $p$ is not equivalent to $p \lor q$, neither is $(p \land (\sim (\sim p \lor q))) \lor (p \land q)$.

6. Prove that proving $(A \Rightarrow B)$ is the same as proving $(\sim B \Rightarrow \sim A)$.

Solution. By truth table:
Since the columns for \( A \implies B \) and \( \neg B \implies \neg A \) are the same, they are equivalent.

7. Prove that \((A \implies B)\) is equivalent to \((\neg B \land A) = False\).

\[\begin{array}{c|c|c|c|c|}
A & B & A \implies B & \neg B & \neg A & \neg B \implies \neg A \\
\hline
T & T & T & F & F & T \\
T & F & F & T & F & F \\
F & T & T & F & T & T \\
F & F & T & T & T & T \\
\end{array}\]

The last column proves that “\((A \implies B)\) is equivalent to \((\neg B \land A)\)” is false.

8. Prove that if \( A = X \lor Y \lor Z \), then if we want to prove \( A \implies B \) then it is enough to prove that \((X \implies B) \land (Y \implies B) \land (Z \implies B)\)

\[\begin{array}{c|c|c|c|c|c|}
A & B & A \implies B & \neg B & \neg A & \neg B \land A \\
\hline
T & T & T & F & F & F \\
T & F & F & T & T & F \\
F & T & T & F & F & F \\
F & F & T & T & T & F \\
\end{array}\]

The last column proves that \((A \implies B)\) is equivalent to \((\neg B \land A)\)” is false.

\[\begin{array}{c|c|c|c|c|c|}
A & B & A \implies B & \neg B & \neg B \land A & (A \implies B) \leftrightarrow (\neg B \land A) \\
\hline
T & T & T & F & F & F \\
T & F & F & T & T & F \\
F & T & T & F & F & F \\
F & F & T & T & T & F \\
\end{array}\]

\[\neg(X \lor Y \lor Z) \lor B \]
\[\neg X \land \neg Y \land \neg Z \lor B \hspace{1cm} \text{DeMorgan’s law}\]
\[\neg X \lor B \land \neg Y \lor B \land \neg Z \lor B \hspace{1cm} \text{distribution}\]
\[(X \implies B) \land (Y \implies B) \land (Z \implies B)\]

Thus, the statements are equivalent.
9. Is the statement form \((p \land q) \lor (\neg p \lor (p \land \neg q))\) a tautology or a contradiction or none.

**Solution.** The statement is a tautology.

**Method 1:** Truth Table

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<td>p</td>
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<td>\neg q</td>
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The last column shows that the statement is a tautology.

**Method 2:** Algebraic Manipulations

\[
\begin{align*}
(p \land q) \lor (\neg p \lor (p \land \neg q)) & \quad \text{distributive rule} \\
(p \land q) \lor ((\neg p) \land (\neg p \lor \neg q)) & \quad \text{distributive rule} \\
(p \land q) \lor (T \land (\neg p \lor \neg q)) & \quad \text{negation rule} \\
(p \land q) \lor (\neg p \lor \neg q) & \quad \text{bound rule} \\
(p \land q) \lor \neg(p \land q) & \quad \text{DeMorgan’s rule} \\
T & \quad \text{negation rule}
\end{align*}
\]

Thus, the statement is a tautology.