1. Write down the following integers in base 7:
   (a) 245
   (b) 98
   (c) 2014

Solution:
   (a) 500
   (b) 200
   (c) 5605

2. What is the representation of the number $[2402]_5$ in base 2?

Solution: $[2402]_5 = 2 \cdot 5^3 + 4 \cdot 5^2 + 0 \cdot 5^1 + 2 \cdot 5^0 = [352]_{10} = [101100000]_2$

3. What is $[111111]_2 + [1]_2$?

Solution: $[111111]_2 + [1]_2 = [1000000]_2$

4. What is $[32132]_4 + [22]_4$?

Solution: $[32132]_4 + [22]_4 = [32220]_4$

5. Let $n$ be an integer. Let the remainder when $n$ is divided by $b$ is $a$. Prove that if $n$ is written in the base $b$ representation as
   $$n = x_0 \cdot b^0 + x_1 \cdot b^1 + \cdots + x_k \cdot b^k,$$
   then $x_0$ must be equal to $a$. 
Proof. We know that the remainder when \( n \) is divided by \( b \) is \( a \). This means that there is some integer \( q \) such that \( n = bq + a \). By substitution, \( bq + a = x_0 \cdot b^0 + x_1 \cdot b^1 + \cdots + x_k \cdot b^k \). If we factor the right side of the equation, we get \( bq + a = x_0 + b(x_1 + \ldots x_k) \). Then we can see that \( q = x_1 + \ldots x_k \) and \( a = x_0 \).

6. What is the maximum integer that can be represented in base 2 using only 10 bits (that is, what is the largest integer which when represented in base 2 has at most length 10 representation).

Solution: \([1111111111]_2 = [1023]_{10}\)

7. (a) Show that if \( a \) and \( b \) are integers in the range 1 through 256, and the sum of \( a \) and \( b \) is also in this range, then

\[
2^9 \leq (2^9 - a) + (2^9 - b) < 2^{10}.
\]

(b) Explain why it follows that the binary representation of \((2^9 - a) + (2^9 - b)\) has a leading term in the \( 2^9 \)th position.

Solution:

(a) We know that \( 1 \leq a + b \leq 256 \). By multiplying -1, we get \(-256 \leq -a - b \leq -1 \). By adding \(2^9 + 2^9 = 1024\), we get \(768 \leq (2^9 - a) + (2^9 - b) \leq 1023\). Since \(2^9 = 512 \leq 768\) and \(1023 < 1024 = 2^{10}, 2^9 \leq (2^9 - a) + (2^9 - b) < 2^{10} \).

(b) From part (a), we see that the smallest that \((2^9 - a) + (2^9 - b)\) can be is \(2^9\) which in binary is a 1 in the \(2^9\)th position, followed by zeros. We also know that \((2^9 - a) + (2^9 - b)\) is strictly smaller than \(2^{10}\), so its binary representation must have a leading term in the \(2^9\)th position.