CSE 20

Lecture 12: Propositional Logic (contd...)
Use of Propositional Logic

- Check the correctness of a statement: whether a sentence/paragraph/proposition is logically correct.
Use of Propositional Logic

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- Make deductions
Use of Propositional Logic

- Check the correctness of a statement: whether a sentence/paragraph/proposition is logically correct.
- Make deductions
- Check if two propositions are equivalent.
If the truth tables of two statement/propositions are identical then the two statement/propositions are equivalent.
Equivalence of Statement: iclicker Question 1

Which of the following is equivalent to $A \implies B$

A. $\neg B \implies \neg A$
B. $(\neg A \land B) = False$
C. $(\neg B \land A) = False$
D. $\neg A \implies \neg B$
E. $(\neg B \lor A)$
To prove statement $B$ from $A$:
Proof Techniques

To prove statement $B$ from $A$:

- **Direct Proof:**
  
  $A \implies B$
Proof Techniques

To prove statement $B$ from $A$:

- Direct Proof:
  
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- Proof by contradiction:
  
  $(\neg B \land A)$ gives a contradiction
Proof Techniques

To prove statement $B$ from $A$:

- **Direct Proof:**
  
  $A \implies B$

- **Proof by contradiction:**
  
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- **Proof by Contrapositive:** $A \implies B$ is same as proving $\neg B \implies \neg A$. 
Equivalence of Statement: iclicker Question 2

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A Boolean expression $f$ having this truth table is:

A. $[\neg p \land \neg q) \lor q] \lor r$
B. $[\neg p \land \neg q) \land q] \land r$
C. $[\neg p \land \neg q) \land \neg q] \land r$
D. $[\neg p \land \neg q) \lor q] \land r$
E. $[\neg p \lor \neg q) \land q] \land r$
The function

\[
\left( \left( p \lor (r \lor q) \right) \land \neg (p \land (\neg q \land \neg r)) \right)
\]

is equal to the function:

A. \( q \lor r \)

B. \( \neg p \lor (r \land q) \)

C. \( (p \lor q) \lor r \)

D. \( (p \lor q) \land \neg (p \lor r) \)

E. \( (p \land r) \lor (p \land q) \)
Rules of Propositional Logic

1. Commutative law:

\[(p \lor q) = (q \lor p) \text{ and } (p \land q) = (q \land p)\]

2. Associative law:

\[(p \lor (q \lor r)) = ((p \lor q) \lor r) \text{ and } (p \land (q \land r)) = ((p \land q) \land r)\]

3. Distributive law:

\[(p \lor (q \land r)) = (p \lor q) \land (p \lor r) \text{ and } (p \land (q \lor r)) = (p \land q) \lor (p \land r)\]

4. De Morgan’s Law:

\[\neg (p \lor q) = (\neg p \land \neg q) \text{ and } \neg (p \land q) = (\neg p \lor \neg q)\]
The function
\[ ( (p \lor (r \lor q)) \land \neg(p \land (\neg q \land \neg r)) ) \]
is equal to the function:

A. \( q \lor r \)
B. \( \neg p \lor (r \land q) \)
C. \( (p \lor q) \lor r \)
D. \( (p \lor q) \land \neg(p \lor r) \)
E. \( (p \land r) \lor (p \land q) \)
Rules of Propositional Logic

Let $p$, $q$ and $r$ be propositions.

1. **Commutative law:**
   
   $$(p \lor q) = (q \lor p) \text{ and } (p \land q) = (q \land p)$$

2. **Associative law:**
   
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   $$\neg (p \lor q) = (\neg p \land \neg q) \text{ and } \neg (p \land q) = (\neg p \lor \neg q)$$
Sets

- Sets

For example:
- Set of names of all students
- Set of letters in the English alphabet
- Set of digits: \{0, 1, \ldots, 9\}

Unordered Sets
Ordered Sets (Also called LIST/STRINGS/VECTORS)

Size of a set is the number of elements in the set.
Sets

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Unordered Sets

Ordered Sets
(Also called LIST/STRINGS/VECTORS)

Size of a set is the number of elements in the set.
Operations on Sets

- **Union, $\cup$.**
  
  $A \cup B$ is the set of all elements that are in $A$ OR $B$.

- **Intersection, $\cap$.**
  
  $A \cap B$ is the set of all elements that are in $A$ AND $B$.

- **Complement, $A^c$ or $\overline{A}$.**
  
  $A^c$ is the set of elements NOT in $A$. 
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  \[ A^c \] is the set of elements NOT in $A$.

- **Cartesian Product.**
Union and Intersection

Union

A ∪ B

A ∩ B

A ∪ B
Rules of Propositional Logic

Let $p$, $q$ and $r$ be propositions.

1. Commutative law:

\[(p \lor q) = (q \lor p) \text{ and } (p \land q) = (q \land p)\]

2. Associative law:

\[(p \lor (q \lor r)) = ((p \lor q) \lor r) \text{ and }\]
\[(p \land (q \land r)) = ((p \land q) \land r)\]

3. Distributive law:

\[(p \lor (q \land r)) = (p \lor q) \land (p \lor r) \text{ and }\]
\[(p \land (q \lor r)) = (p \land q) \lor (p \land r)\]

4. De Morgan’s Law:

\[\neg(p \lor q) = (\neg p \land \neg q) \text{ and } \neg(p \land q) = (\neg p \lor \neg q)\]
Rules of Set Theory

Let $p$, $q$ and $r$ be sets.

1. Commutative law:
   \[(p \cup q) = (q \cup p) \text{ and } (p \cap q) = (q \cap p)\]

2. Associative law:
   \[(p \cup (q \cup r)) = ((p \cup q) \cup r) \text{ and } (p \cap (q \cap r)) = ((p \cap q) \cap r)\]

3. Distributive law:
   \[(p \cup (q \cap r)) = (p \cup q) \cap (p \cup r) \text{ and } (p \cap (q \cup r)) = (p \cap q) \cup (p \cap r)\]

4. De Morgan’s Law:
   \[(p \cup q)^c = (p^c \cap q^c) \text{ and } (p \cap q)^c = (p^c \cup q^c)\]
Set Theory and Propositional Logic is two mathematical language which follow very similar rules.
If $A$ and $B$ are two sets such that size of $A$ is 10 and size of $B$ is 12. If size of $A \cup B$ is 20 what is size of $A \cap B$.

A 2
B 3
c 10
D 22
E Can’t be said.