Recall that a Turing machine $M$ decides a language $L$ if it accepts all strings in $L$ and rejects all strings not in $L$. In particular, $M$ always terminates. In problems 1 and 2, you should use jflap to design and test your machine on several inputs. Notice that Turing machines in jflap do not have an explicit reject state, and they reject the input when they get stuck.

**Problem 1**

Give a TM that decides the language $L = \{wxw: w \in \{0,1\}^*\}$. E.g., $01001x01001$ should be accepted, but $111x1$ should be rejected.

**Problem 2**

Give a TM that on input a sequence $1^n$ of $n$ ones, decides if $n$ is a prime number. (The problem can be solved with 25 states or fewer.) *Hint:* design a Turing machine that appends “$x1$” to the input, so that the tape content is $1^n x1$. Then, repeatedly do the following: append one more 1 at the end to obtain a string $1^n x1^k$ for $k \geq 2$. Check if $n$ is a multiple of $k$. If not, keep incrementing $k$ by appending 1s at the end. If at some point $n$ is a multiple of $k$, check if $k = n$.

**Problem 3**

A function $f: \Sigma^* \to \Sigma^*$ is called “Turing computable” if there is a Turing machine $M$ that on input $w \in \Sigma^*$, it always terminates, and upon termination the content of the tape is $f(w)$.

Recall that the inverse image of a set $A \subseteq \Sigma^*$ under a function $f$ is the set $f^{-1}(A) = \{x \in \Sigma^*: f(x) \in A\}$. Prove that decidable languages are closed under preimage, i.e., if $L \subseteq \Sigma^*$ is decidable and $f$ is a Turing computable function, then $f^{-1}(L)$ is also decidable.

**Problem 4**

Now consider closure under image: is it true that if $L$ is a decidable language, and $f$ is a computable function, then the language $f(L) = \{f(x): x \in L\}$ is decidable? Prove or disprove.