CSE105 (spring 2013): Homework 4

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Due on Monday, May 6, 2013.

Problem 1

In class we claimed, without a proof, that if $L$ is a regular language and $f$ is a function computed by an NFST, then $f(L)$ is regular. Some doubts about the validity of the statement were raised in class. Prove that the claim is correct! [Hint: Prove that any DFA $M$ and NFST $T$ can be combined into a GNFA $N$ such that the language of $N$ is precisely $f_T(L(M))$.]

Problem 2

Problem 1.46 (a) and (d) from Sipser.

Problem 3

Problem 1.55 (c), (e), (f), (i) and (j) from Sipser.

Problem 4

Let $L$ be the language of all strings over the alphabet $\{a, b\}$ containing an unequal number of $a$’s and $b$’s. Prove (using the pumping lemma and/or the closure properties of regular languages) that $L$ is not regular.

Problem 5

In Homework 1 (problem 3), you gave a finite automaton for the the language $L$ of all binary strings $x_1y_1x_2y_2\ldots x_ny_n$ of even length $2n$ such that $x = x_1\ldots x_n$ and $y = y_1\ldots y_n$ are the binary representation of two numbers that add up to $x + y = 2^n$. This proves that the language $L$ is regular. In this problem you will see that the regularity of the language depends on the specific input representation, and that seemingly small changes in the encoding can make the language nonregular.

Consider the language $L$ defined as above, except that the two inputs are concatenated one after the other. In other words, $L$ is the set of strings of the form $x_1\ldots x_ny_1\ldots y_n$, where $x_i, y_i \in \{0, 1\}^*$, and the numbers with binary representation $x_1\ldots x_n$ and $y_1\ldots y_n$ add up to $2^n$. Prove (using the pumping lemma and/or the closure properties of regular languages) that this language is not regular.