1. Consider a Turing machine model with a single tape but with $k$ tape heads that can move independently. Show that for every time $T(n) \geq n$, $TIME_{k\text{-tape}}(T(n))$ for $k$-tape Turing Machines is contained in the $TIME_{k\text{heads},1\text{tape}}(T(n))$.

2. Consider the language $AddFacts = \{x'' + ''y'' = ''z'' \mid x, y, z \text{ are binary integers and } x + y = z\}$. Prove that $AddFacts$ is in linear time on a multitable TM, but requires quadratic time on a one-tape TM.

3. If $L$ is a language, $L^*$ is the set of all strings $x$ so that, for some $k \geq 0$, $x = x_1 \circ x_2 \circ x_3 \ldots \circ x_k$ where each $x_i \in L$ and $\circ$ denotes concatenation. Show that if $L \in P$, then $L^* \in P$. Hint: dynamic programming.

4. Consider a language of pairs of strings $U$. For each $x$, let $U_x$ be the language of all $y$ so that $(x, y) \in U$. Can there be a recursive language $U$ so that every recursive language $L$ can be written as $U_x$ for some $x$? Prove your answer.

5. Give an example of a language $L$ that is neither R.E. nor co-R.E. Prove your answer correct.