CSE 200 Final Exam
Due Wed. June 13 at 11:59 PM (OK, I won’t actually check until I get in in
Thursday morning. But all exams must be turned in before I arrive in my
office.)

Answer four out of five questions with an informal, but complete, proof. You
may not discuss this exam with anyone except myself and Fjola, whether taking
the course or not. Each question has equal weight, but some are more difficult
than others. You may cite without proof any result from the Arora-Barak or
Sipser text or proved in class. In particular, you can use without proof the
NP-completeness of any problem proved NP-complete in class, in the two texts,
or on the homeworks, including: SAT, 3-SAT, Independent Set, Clique, Vertex
Covering, 3-coloring, Hamiltonian Circuit, and Subset Sum. However, you may
not use the list of NP-complete problems in the appendix of Garey and Johnson
without proof. If you turn in all five, the hardest one to grade will be omitted.

Computability Let $x \circ y$ denote the concatenation of strings $x$ and $y$. For a
language $L \in \{0,1\}^*$, let $\text{pref}(L) = \{x \in \{0,1\}^* \mid \exists y \in \{0,1\}^*, x \circ y \in L\}$. Say that a class $C$ is closed under prefixes if whenever $L \in C$, then $\text{pref}(L) \in C$. Is the class of recursive (aka computably decidable) lan-
guages closed under prefixes? The class of R.E. (aka computably enumer-
able) languages?

NP-Completeness: Modular equations Prove that the following problem
is NP-complete:

Problem: Boolean solution for mod-5 equations.
Instance: A set of $m$ equations $E_1, \ldots, E_m$ in $n$ variables $x_1, \ldots, x_n$, where
each equation $E_j$ is of the form $\sum_{i=1}^{n} a_{i,j} x_i = c_j (\text{mod } 5)$ where the
$a_{i,j}, c_j \in \{0, \ldots, 4\}$.
Solution space: A Boolean value $b_i \in \{0,1\}$ for each $x_i$.
Constraints: Each equation is true for the assignment, i.e., $(\sum_{i} a_{i,j} b_i) \text{mod } 5 =
c_j$ for each $j$.
Objective: Decide whether there is a Boolean solution meeting the con-
straints.

NP-Completeness: Path with Minimum Distinct Labels Prove that the
following problem is NP-complete:

Problem: Path with Minimum Distinct Labels
Instance: A directed graph $G$, a set of labels $L$, and a label $l(e) \in L$ for
each edge $e$. A start node $s \in V$ and a finish node $t \in V$. A budget
$B$.
Solution space: A path $p$ from $s$ to $t$ in $G$. 

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Objective function: Minimize the number of distinct labels in \( p \), i.e.,
\[
\text{minimize } |\{ l(e) | e \in p \}|
\]

Objective: Decide whether there is a path that uses at most \( B \) labels.

Circuits and the polynomial-time hierarchy: Remember, \( \text{EXP} = \bigcup_k \text{TIME}(2^{n^k}) \),
and \( \text{P/poly} \) is the set of functions with polynomial-sized circuit families.
Prove the following theorem of Karp and Lipton: if \( \text{EXP} \subseteq \text{P/poly} \), then
\( \text{EXP} = \Sigma^p_2 \).

(Hint: define a new language \( L' \) in \( \text{EXP} \) so that whether \( (x, i, T) \in L' \)
codes the \( i \)'th cell of the configuration of an \( \text{EXP} \) machine on input \( x \) at
time \( t \). Use \( L' \in \text{P/poly} \) to put \( L \) in \( \Sigma^p_2 \).)

Space complexity Show that \( \text{P}^{\text{PSPACE}} = \text{NP}^{\text{PSPACE}} \).