DISCUSSION OF FRIDAY 25\textsuperscript{th}:  RECURSION

**Binary Search Trees (Recursion)**

Given a set of elements, we want to order them quickly and efficiently.

Example: \( L = \{14, 12, 8, 3, 17, 13, 7, 29, 5, 10, 9\} \)

1. Take any element as a "root". E.g.: 12

Each node has max 2 "children". On the left, one that is smaller, on the right, one that is bigger than the node itself.

- A bin. tree is SORTED if every node in the tree is larger than (or equal to) its left child, and smaller than (or equal to) its right child.
- Equal nodes can go either on the right or on the left (but it has to be consistent).
- Searching a binary tree (to find a value) can be done recursively ("tree traversal"). You can code a bin. tree through recursion.
EXAMPLE RECURSIVE FUNCTION

\[
\begin{aligned}
F(0) &= 0 \\
F(1) &= 1 \\
F(n) &= (F(n-1))^n + (F(n-2))^{-(n-1)}, \quad n > 1
\end{aligned}
\]

? \rightarrow F(5)

\[
\begin{aligned}
F(2) &= 1^2 + 0^{-1} = 1 \\
F(3) &= 1^3 + 1^{-2} = 2 \\
F(4) &= 2^4 + 1^{-3} = 2^4 + 1 = 17 \\
F(5) &= (17)^5 + (2)^{-4} = 17^5 + \frac{1}{2^4} = 89,521.9625
\end{aligned}
\]
The Fibonacci Sequence

• “Facts” about rabbits
  – Rabbits never die
  – A rabbit reaches maturity exactly two months after birth, that is, at the beginning of its third month of life
  – Rabbits are always born in male-female pairs

• At the beginning of every month, each mature male-female pair gives birth to exactly one male-female pair

Problem How many pairs of rabbits are alive in month n?

Solution: Recurrence relation: rabbit(n) = rabbit(n-1) + rabbit(n-2)

Base cases rabbit(2), rabbit(1)
Recursive definition

1 if n is 1 or 2
rabbit(n) = rabbit(n-1) + rabbit(n-2) if n > 2

Fibonacci sequence: The series of numbers rabbit(1), rabbit(2), rabbit(3), and so on

Example: Consider the recurrence relation a_n = 2a_{n-1} – a_{n-2} for n = 2, 3, 4, ...

  • Is the sequence {a_n} with a_n=3n a solution of this recurrence relation?

Solution: If a_n=3n then it must be that: a_{n-1}=3(n-1) and a_{n-2}=3(n-2)
For n >= 2 we see that 2a_{n-1} – a_{n-2} = 2(3(n – 1)) – 3(n – 2) = 3n = a_{n}.
Therefore, {a_n} with a_n=3n is a solution of the recurrence relation.

  • Is the sequence {a_n} with a_n=5 a solution of the same recurrence relation?

Solution: For n>= 2 we see that 2a_{n-1} – a_{n-2} = 2*5 - 5 = 5 = a_{n}.
Therefore, {a_n} with a_n=5 is also a solution of the recurrence relation.