Generalized Hough Transform, line fitting
Introduction to Computer Vision
CSE 152
Lecture 11

Canny Edge Detector
1. Smooth image by filtering with a Gaussian
2. Compute gradient at each point in the image.
3. At each point in the image, compute the direction of the gradient and the magnitude of the gradient.
4. Perform non-maximal suppression to identify candidate edgels.
5. Trace edge chains using hysteresis thresholding.

So, to detect corners
• Filter image with a Gaussian.
• Compute the gradient everywhere.
• Move window over image and construct C over the window.
• Use linear algebra to find $\lambda_1$ and $\lambda_2$.
• If they are both big, we have a corner.
  1. Let $e(x,y) = \min(\lambda_1(x,y), \lambda_2(x,y))$
  2. $(x,y)$ is a corner if it’s local maximum of $e(x,y)$ and $e(x,y) > \tau$

Parameters: Gaussian std. dev, window size, threshold

Corner Detection
What is region like if:
1. $\lambda_2 = 0$?
2. $\lambda_2 = 0$?
3. $\lambda_1 = 0$ and $\lambda_2 = 0$?
4. $\lambda_1 > 0$ and $\lambda_2 > 0$?

What to do with edges?
• Segment linked edge chains into curve features (e.g., line segments).
• Group unlinked or unrelated edges into lines (or curves in general).
• Accurately fitting parametric curves (e.g., lines) to grouped edge points.

Announcements
• Assignment 2: Due Friday
• Midterm: Thursday, May 10 in class, We’ll talk about it at the end of class
Hough Transform [Patented 1962]

Finding lines in an image

- A line in the image corresponds to a point in Hough space
- To go from image space to Hough space:
  - Given a set of points \((x, y)\), find all \((m, b)\) such that \(y = mx + b\)

**Hough transform algorithm**

- Typically use a different parameterization
  
  \[ d = \rho \cos \theta + y \sin \theta \]

  - \(d\) is the perpendicular distance from the line to the origin
  - \(\theta\) is the angle this perpendicular makes with the x-axis
  - Why?

- Basic Hough transform algorithm
  1. Initialize \(H(d, \theta) = 0\); \(H\) is called accumulator array
  2. For each edge point \((x, y)\) in the image
     - For \(\theta = 0\) to 180
       - \(H(d, \theta) += 1\)
  3. Find the value(s) of \((d, \theta)\) where \(H(d, \theta)\) is maximum
  4. The detected line in the image is given by
     
     \[ d = \rho \cos \theta + y \sin \theta \]

- What’s the running time (measured in \(N\) votes)?

Hough Transform: 20 colinear points

Image | Accumulator
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- R, \(\theta\) representation of line
- Maximum accumulator value is 20

Hough Transform: Random points

Image | Accumulator
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- R, \(\theta\) representation of line
- Maximum accumulator value is 4
Hough Transform: “Noisy line”

Image

Accumulator

• r, θ representation of line
• Maximum accumulator value is 6

Hough Transform for Curves
(Generalized Hough Transform)

The H.T. can be generalized to detect any curve that can be expressed in parametric form:

- Y = f(x, a₁, a₂, ..., aₚ)
- Or g(x, y, a₁, a₂, ..., aₚ) = 0
- a₁, a₂, ..., aₚ are the parameters
- The parameter space is p-dimensional
- The accumulating array is LARGE!

Example: Finding circles

Equation for circle is

\[(x - x_c)^2 + (y - y_c)^2 = r^2\]

Where the parameters are the circle’s center \((x_c, y_c)\) and radius \(r\).

Three dimensional generalized Hough space.

TEM Image of Keyhole Limpet
Hemocyanin with detected particles

Processing in Stage 1 for KLH

- Canny edge detection.
- A sequence of ordered Hough transforms (HT’s) is applied in order from the computationally simplest one to the most complex one.
- Edges covered by the detected shapes are removed immediately from edge images following the application of the last HT.

3D Maps of KLH

(a)  (b)  (c)  (d)

FIG. Three-dimensional maps of KLH at a resolution of 23.5 Å reconstructed using particles extracted either manually or automatically as described in the text. (a), (b) The side- and top- view of a 3D map reconstructed from a set of 1042 manually selected particle images. (c), (d) The side- and top- view of a 3D map from a set of automatically extracted 1243 particle images.
Picking KLH Particles in Stage 1

Zhu et al., IEEE Transactions on Medical Imaging, In press, 2003

Line Fitting

Given \( n \) points \((x_i, y_i)\), estimate parameters of line \( ax_i + by_i + d = 0 \) subject to the constraint that \( a^2 + b^2 = 1 \).

Cost Function:
Sum of squared distances between each point and the line \((x_i, y_i)\)

1. Minimize \( E \) with respect to \((a, b, d)\).
   - \( E(a, b, d) = \sum_{i=1}^{n} (ax_i + by_i + d)^2 \)
   - \( (x, y) \) is the mean of the data points

2. Substitute \( d \) back into \( E \)
   - \( n = (a, b)^T \)

3. Minimize \( E = n^T S n + n^T d \) with respect to \( a, b \) subject to the constraint \( n^T n = 1 \).
   - Note that \( S \) is given by
   - \( S = \mu^T \mu = \left( \begin{array}{ccc} \sum x_i^2 - n \bar{x}^2 & \sum x_i y_i - n \bar{x} \bar{y} \\ \sum x_i y_i - n \bar{x} \bar{y} & \sum y_i^2 - n \bar{y}^2 \end{array} \right) \)
   - And it’s a real, symmetric, positive definite

4. This is a constrained optimization problem in \( n \). Solve with Lagrange multiplier
   - \( L(n) = n^T S n - \lambda (n^T n - 1) \)
   - Take partial derivative (gradient) w.r.t. \( n \) and set to 0.
   - \( \nabla L = 2S n - 2\lambda n = 0 \)
   - or
   - \( S n = \lambda n \)
   - \( n = (a, b) \) is an Eigenvector of the symmetric matrix \( S \) (the one corresponding to the smallest Eigenvalue).

5. \( d \) is computed from Step 1.

Line Fitting – Finished

Midterm
Thursday, May 5

- In class
- Full period
- Coverage – everything up to this point including readings
- “Cheat sheet” – you can prepare a one sided sheet of notes. It must be hand written.
  (After the midterm, save your sheet since you can use the other side for the final).
- No calculators.
Incomplete list of topics covered...

• Human visual system
  – Physiology – from eye to brain
  – Phenomenological
  – Function
• Camera models
• Factors in producing images
• Projection models
  – Perspective
  – Orthographic
• Homogenous Coordinates, Vanishing points
• Lenses, Distortion
• Sensors
• Quantization/Resolution
• Illumination
• Reflectance
  – BRDF
  – Lambertian
  – Specular
  – Phong
• Color
  – Light Spectrum
  – Reflectance, source
  – Sensor response
  – Color spaces
  – Chromaticity, YUV, RGB

Topics cont.

• Binary Vision
  – Thresholding
  – Neighborhoods
  – Connected component exploration
  – Features, moments
• Noise
  – Additive, Gaussian noise
• Filtering, linear, convolution with Kernel
  – Averaging/smoothing
  – Sharpening
  – Derivatives
  – Gaussian filter
  – Seperability
• Edges & Edge detection
• Edge sources
• Canny
  – Gaussian derivatives
  – Magnitude, orientation
  – Non-maximal suppression
  – Linking/thresholding
• Hough Transform
• Generalized Hough transform
• Line fitting