Matrix Vector Identities

In the following questions, $A$ is a $d \times d$ matrix, and $x$ and $y$ are $d \times 1$ vectors. Prove the following matrix-vector identities:

1. $x^\top Ax = \sum_{i=1}^{d} \sum_{j=1}^{d} A_{ij} x_i x_j$.
2. $x^\top y = \langle x, y \rangle$.
3. $x^\top x = \|x\|^2$.
4. $xy^\top$ is a $d \times d$ matrix, whose $(i, j)$-th entry is $x_i y_j$.
5. Let $B = [b_1, b_2]$, where $b_1$ and $b_2$ are $d \times 1$ column vectors. Then, $AB = [Ab_1, Ab_2]$.
6. Let $C = [c_1, c_2]$, where $c_1$ and $c_2$ are $d \times 1$ column vectors. Then,

$$C^\top B = \begin{bmatrix} c_1^\top b_1 & c_1^\top b_2 \\ c_2^\top b_1 & c_2^\top b_2 \end{bmatrix}$$