Problem One

\[ 24 = 0000011000 \]
\[ 17 = 0000010001 \]

(3 points)

\[ -17 = 1111101110 \]

(3 points)

\[
\begin{array}{c}
\phantom{-}0000011000 \\
+ \phantom{0}1111101110 \\
\hline \phantom{-}0000000110 + 1 \\
\end{array}
\]

\[ = 0000000111 \] (4 points)

Problem Two

2.1

- nine’s complement: \( 10^n - x - 1 \) (2 points);
- ten’s complement: \( 10^n - x \) (2 points);

2.2

- nine’s complement: \( 10^5 - 47 - 1 = 99952 \) (2 points);
- nine’s complement: \( 00031 + 99952 = 99983 \) (1 points);
- ten’s complement: \( 10^5 - 47 = 99953 \) (2 points);
- ten’s complement: \( 00031 + 99953 = 99984 \) (1 points);
3 Problem Three

\[(a + bc)'(a'b + c)\]
\[= a'(b' + c)(a'b + c)\]
\[= (a'b' + a'c)(a'b + c)\]
\[= a'a'b' + a'b'c + a'a'bc + a'cc\]
\[= a'c(b' + b + 1)\]
\[= a'c\]

Policy: 10 points if all steps are correct, otherwise 0.

4 Problem Four

Please the image on the website.

5 Problem Five

- (5.1): \(a_1 = 0, a_2 = 1, a_3 = 2\) (5 points);
- (5.2): \(a_n = a_{n-2} + 2a_{n-3}\) (5 points);
- (5.3): (5 points)

\[x^3 - x - 2 = 0\]

\[\Rightarrow x_1 = 1.52138\]
\[x_2 = -0.76069 + 857874i\]
\[x_3 = -0.76069 - 857874i\]
\[a_n = C_1 x_1^n + C_2 x_2^n + C_3 x_3^n\]

6 Problem Six

Let \(s_i\) be the number of games played up to day \(i\) (3 points) and \(t_i = s_i + 9\) (3 points). There are \(18 + 18 = 36\) numbers (pigeons, 2 points) in sequence \(s\) and \(t\) whose values are in the range \([1,34]\) (holes, 2 points). Hence, by pigeon-hole principle, there exist \(i < j\) such that \(s_j = t_i = s_i + 9\) and exactly 9 games were played in the period from day \(i + 1\) to day \(j\).

7 Problem Seven

Let \(s_i = x_1 + x_2 + \cdots + x_i\) be the partial sum of sequence \((x_1, x_2, \ldots, x_n)\) (3 points). Let \(t_i = s_i \mod n\) (3 points). If there exists \(i\) such that \(t_i = 0\), then \(\{1, 2, \ldots, i\}\) is the set desired. Now we assume \(t_i \neq 0\) and \(t_i\) belongs to the range \([1, n - 1]\) (holes, 2 points). Since we have \(n\) numbers in sequence \(t\) (pigeons, 2 points), there exist \(i < j\) such that \(t_i = t_j\) by pigeon-hole principle. In this case, \(\{i + 1, i + 2, \ldots, j\}\) is the set desired.
8 Problem Eight

8.1 (5 points)

\[
\begin{array}{cccc|c|c}
E & D_1 & D_0 & A & Y \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 1 & 0 \\
1 & 1 & 0 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 \\
\end{array}
\]

8.2

Policy: 5 points if the K-map is correct

8.3

\[
A = (E' + D_1' D_0')' = E(D_1 + D_0)
\]

\[
Y = (E' + D_1')' = ED_1
\]

Policy: 5 points if both expressions are correct.

9 Problem Nine

Please the image on the website.