1. (two’s and one’s complements) Show the operation of $24 + (-17)$ in one’s complement of binary number system. Assume that each binary number is represented with 10 bits.

2. (number systems) We have defined and learned the idea of two’s and one’s complements for n-bit binary numbers.

2.1. Define the complements (corresponding to two’s and one’s) using an n-digit system with base 10.

2.2. Show the arithmetic of $x - y$ where $x = 31_{10}$ and $y = 47_{10}$ in the complement representations (corresponding to two’s and one’s) using a 5-digit system with base 10.

3. (Boolean algebra) Express Boolean function $E(a, b, c) = (a + bc)'(a'b + c)$ in sum-of-products form using Boolean algebra laws and theorems.

4. (product of sums) Express Boolean function $E(a, b, c) = abc + (a' + b)(b + c)'$ in product-of-sums form using Boolean algebra laws and theorems.

5. (recursive function) A frog knows 3 jumping styles ($A, B, C$). With style $A$ the frog jumps forward by 2 feet, and with styles, $B, C$, the frog jumps forward by 3 feet. Let $a_i$ denote the number of ways to jump over a total distance of $i$ feet.

5.1. Write the values of $a_1, a_2, a_3$?

5.2. Derive the recursive formula of $a_n$?

5.3. Find the solution of the recursion.

6. (pigeonhole principle) A team plays 25 games in a 18-day period, and plays at least one game a day. Show that there is a period of days in which exactly 9 games were played.

7. (pigeonhole principle) Let $X = \{x_i\}$ be a set of $n$ positive integers. Show that we can find a nonempty subset of $X$ so that the sum of the integers in the subset is divisible by $n$.

8. (applications of Boolean algebra) A priority encoder is described by its input, output and operation.

   Input: Three binary bits $E$ (Enable), $D_1, D_0$.

   Output: $A, Y$.

   If $E = 0$, then $A = 0, Y = 0$;
   Else if $D_1 = 1$, then $A = 1, Y = 1$;
   Else if $D_0 = 1$, then $A = 1, Y = 0$;
   Else $A = 0, Y = 0$.

8.1. Write the truth table of the priority encoder.

8.2. Write the Karnaugh maps of the outputs and derive minimal expressions in product of sums format.

9. (applications of Boolean algebra) A four bit counter has four binary inputs ($a, b, c, d$) and three binary outputs ($s_2, s_1, s_0$). The binary code of the output represents the number of "1"s in the input. For example, if $(a, b, c, d) = (1, 0, 1, 1)$ then $(s_2, s_1, s_0) = (0, 1, 1)$.

9.1. Write the truth table of the three outputs.

9.2. Write the Karnaugh maps of the outputs and derive minimal expressions in sum of products format.