1. Consider a machine model with a single two-dimensional tape, where locations are indexed by pairs $(x, y)$ of positive integers, where each location can store a symbol from a finite set $\Sigma$, and the input is written on positions $(1,1), \ldots, (n,1)$. The tape head starts at position $(1,1)$, and in one step, the machine can move the tape head to the left one (decrementing $x$), to the right one (incrementing $x$), up one (incrementing $y$) or down 1, (decrementing $y$). Show that the set of languages accepted in polynomial-time on such a 2D-TM is the same as for a normal TM.

2. Remember that for multi-tape TM’s, the input tape is usually considered read-only, and we only count the amount of the other tapes used to measure the amount of memory $S(n)$ an algorithm uses on inputs of length $n$. (Thus, it makes sense for $S(n) < n$.) For the language $L = \{x0^n x|x = n\}$ from class, prove that any $k$-tape TM algorithm that decides membership in $L$ in time $T(n)$ and memory $S(n)$ must satisfy the time-space tradeoff $T(n)S(n) \in \Omega(n^2)$.

3. Show that a function $f(x)$ from $\{0,1\}^*$ to $\{0,1\}^*$ is in $FP$ (functions computable in polynomial time) if and only if there is a $k$ so that $|f(x)| \in O(|x|^k)$ and the language $\{(x, i, b)|i \leq |f(x)|$ and the $i$’th bit of $f(x)$ is $b\}$ is in $P$.

4. Let $L = \{< M, w > | M$ is a Turing Machine program so that there is an input $y \in \{0,1\}^*$ so that $M$ halts on $y$ in at most $100|y|$ steps and $M(y) = w \}$ Is $L$ recursive? Is $L$ recursively enumerable? Is $L$ co-r.e.?

5. Give an example of a language $L$ that is neither R.E. nor co-R.E. Prove your answer correct.