CSE 200 Final Exam

Due Wed. June 9 at 1 AM (OK, I won't actually check until I get in in the morning. But all exams must be turned in before I arrive in my office.)

Answer four out of five questions with an informal, but complete, proof. You may not discuss this exam with anyone except myself and William, whether taking the course or not. Each question has equal weight, but some are more difficult than others. You may cite without proof any result from the Arora-Barak or Sipser text or proved in class. In particular, you can use without proof the NP-completeness of any problem proved NP-complete in class, in the two texts, or on the homeworks, including: SAT, 3-SAT, Independent Set, Clique, Vertex Covering, 3-coloring, Hamiltonian Circuit, and Subset Sum. However, you may not use the list of NP-complete problems in the appendix of Garey and Johnson without proof.

Computability Give an example of a language that is not decideable even with an oracle for the Halting Problem, i.e., that is not Turing reducible to the Halting Problem. Prove that your answer is correct.

NP-Completeness Prove that the following problem is NP-complete:

Problem: Machine budget

Instance: A bipartite graph $G = (M, J, E)$, where the nodes in M represent types of machines, the nodes in J represent types of jobs, and the edges represent that the machine is capable of performing the job. For each machine m, an integer price $p_m > 0$. A positive integer B representing the total budget.

Solution space: A subset $S \subset M$ of machines to purchase.

Constraints: For each $j \in J$, there must me some $m \in S$ that can perform job j, i.e., so that $(m, j) \in E$. The total price $\sum_{m \in S} p_m$ of purchased machines must be at most B.

NP-Completeness Prove that the following problem is NP-complete:

Problem: Balanced 3-coloring

Instance: An undirected graph G

Solution space: A three-coloring χ of the nodes of G

Constraints: The coloring must be a valid 3-coloring, i.e., $\chi(x) \neq \chi(y)$ for every edge $\{x, y\}$, and the number of nodes of each color must be identical.

NP, polynomial hierarchy, and probabilistic computation Prove that, if $NP \subseteq BPP$, then $\Sigma_i \subseteq BPP$ for all i. (Hint: use the fact that the error probability for a probabilistic algorithm can be reduced to be exponentially small.)

Space complexity Remember that k-SAT is the satisfiability problem restricted to CNF formulas with clause sizes at most k. Prove that 2-SAT is NL-complete (under deterministic logspace mapping reductions). (Hint: at least half of the problem is to prove that 2-SAT is in NL. Also, you may use any facts about NL mentioned in class without proof.)