March 29, 2010

1. Let $Reg_k$ be the class of languages accepted by a deterministic finite automaton with at most $k$ states. Prove that for every $k > 0$, $Reg_k$ is a strict subset of $Reg_{k+1}$.

2. Let $f$ be a non-decreasing, positive integer-valued function over the positive integers. Prove that if $f(2n) \in O(f(n))$, then there is a $k$ so that $f(n) \in O(n^k)$. Is the converse always true? Prove it or give a counter-example.

3. a. Prove that any number $n$ so that $n \mod 4 = 3$ has a prime factor $p$ with $p \mod 4 = 3$. b. Prove that there are infinitely many primes $p$ with $p \mod 4 = 3$.

4. In your favorite programming language, write a program that takes no input and prints itself (its own code). Your program may not make system calls.