

CSE 200  
Computability and Complexity  
Calibration Homework  
Due April 7

March 29, 2010

1. Let  $Reg_k$  be the class of languages accepted by a deterministic finite automaton with at most  $k$  states. Prove that for every  $k > 0$ ,  $Reg_k$  is a strict subset of  $Reg_{k+1}$ .
2. Let  $f$  be a non-decreasing, positive integer-valued function over the positive integers. Prove that if  $f(2n) \in O(f(n))$ , then there is a  $k$  so that  $f(n) \in O(n^k)$ . Is the converse always true? Prove it or give a counter-example.
3. a. Prove that any number  $n$  so that  $n \bmod 4 = 3$  has a prime factor  $p$  with  $p \bmod 4 = 3$ . b. Prove that there are infinitely many primes  $p$  with  $p \bmod 4 = 3$ .
4. In your favorite programming language, write a program that takes no input and prints itself (its own code). Your program may not make system calls.