Problem 1 (9 points)

Consider the following while loop:

while (m>0 and m<1000) {
    m=m-1;
    a=a+3;
}

For each of the following predicates, say if it is a valid loop invariant or not, and prove your answer. (As a reminder, a loop invariant is a predicate P such that if P if satisfied before the execution of the loop, then P is satisfied at every subsequent iteration, and upon exiting the loop.)

1. $a = 3m + 1$
2. $a = m + 3$
3. $a + m$ is even

Problem 2 (8 points)

Consider the following program fragment

i=0;
sum=0;
while (i<n) {
    sum=sum+A[i];
    i=i+1;
}

where $n$ is a positive integer and $A$ is an array of $n$ integers $A[0], \ldots, A[n-1]$. Prove, using an appropriate loop invariant, that at the end of the while loop the variable $sum$ equals the sum $\sum_{j=0}^{n-1} A[j]$ of all values stored in the array.

Problem 3 (8 points)

Let $f : X \rightarrow Y$, $g : Y \rightarrow Z$ and defined the function composition $h = g \circ f : X \rightarrow Z$. Prove or disprove each of the following assertions:

1. If $h$ is one-to-one, then either $f$ or $g$ must be one-to-one
2. If $h$ is a bijection, then both $f$ and $g$ must be surjective
Problem 4 (8 points)

Consider the following sets: $A = \{0, 1, 2\}$, $B = \{a, b\}$. Answer the following questions:

1. What is the size of $A \times B$?
2. What is the size of $A^B$?
3. What is the number of binary relations over the set $A$?
4. What is the number of surjective functions from $A$ to $B$?
5. What is the number of injective functions from $B$ to $A$?
6. What is the number of subsets of $A \cup B$ of size at most 3?
7. There exist two functions $f : A \to B$ and $g : B \to A$ such that $f \circ g$ is injective?
8. There exist two functions $f : A \to B$ and $g : B \to A$ such that $f \circ g$ is surjective?

(For this problem, no proofs are required. You are only asked to provide the correct answers.)