Announcements

- Read Trucco & Verri: pp. 15-40
- HW1 will be on website tomorrow or Saturday.
- Irfanview: http://www.irfanview.com/ is a good windows utility for manipulating images. Try xv for linux.

Pinhole Camera: Perspective projection

- Abstract camera model - box with a small hole in it

Geometric Aspects of Perspective Projection

- Points project to points
- Lines project to lines
- Angles & distances (or ratios) are NOT preserved under perspective
- Vanishing point

The equation of projection

Cartesian coordinates:
- We have, by similar triangles, that \((x, y, z) \rightarrow (f \frac{x}{z}, f \frac{y}{z}, -f)\)
- Ignore the third coordinate, and get

Euclidean \rightarrow Homogenous \rightarrow Euclidean

In 2-D
- Euclidean \rightarrow Homogenous: \((x, y) \rightarrow \lambda (x,y,1)\) (can just take \(\lambda = 1\))
- Homogenous \rightarrow Euclidean: \((x, y, z) \rightarrow (x/z, y/z)\)

In 3-D
- Euclidean \rightarrow Homogenous: \((x, y, z) \rightarrow \lambda (x,y,z,1)\) (can just take \(\lambda = 1\))
- Homogenous \rightarrow Euclidean: \((x, y, z, w) \rightarrow (x/w, y/w, z/w)\)
The camera matrix

Turn $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} f \frac{x}{z} \\ f \frac{y}{z} \end{pmatrix}$

into homogenous coordinates
- HC’s for 3D point are $(X,Y,Z,1)$
- HC’s for point in image are $(U,V,W)$

Affine Camera Model

- Take Perspective projection equation, and perform Taylor Series Expansion about (some point $(x_0, y_0, z_0)$).
- Drop terms of higher order than linear.
- Resulting expression is called affine camera model.
- Properties
  - Pts. map to pts, lines map to lines
  - Parallel lines map to parallel lines (no vanishing point – at infinity)
  - Ratios of distance/angles preserved

Orthographic projection

Start with affine camera model, and take Taylor series about $(x_0, y_0, z_0) = (0, 0, z_0)$ – a point on optical axis

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$

Depth (z) is lost

What if camera coordinate system differs from object coordinate system

Euclidean Coordinate Systems

Coordinate Changes: Pure Translations
No rotation (e.g., $i_k = i_o$ etc)

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \bar{OP} \hat{i} + \bar{OP} \hat{j} + \bar{OP} \hat{k}$

$\bar{OP} = xi + yj + zk$
A rotation matrix $R$ has the following properties:

- Its inverse is equal to its transpose $R^{-1} = R^T$.
- Its determinant is equal to 1: $\text{det}(R) = 1$.

Or equivalently:

- Rows (or columns) of $R$ form a right-handed orthonormal coordinate system.

Rotation: Homogenous Coordinates

- About $z$ axis

\[
\begin{bmatrix}
    x' \\
    y' \\
    z' \\
    1
\end{bmatrix} =
\begin{bmatrix}
    \cos \theta & -\sin \theta & 0 & 0 \\
    \sin \theta & \cos \theta & 0 & 0 \\
    0 & 0 & 1 & 0 \\
    0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
    x \\
    y \\
    z \\
    1
\end{bmatrix}
\]

Note: $z$ coordinate doesn’t change after rotation.

Rotation

- About $x$ axis:

\[
\begin{bmatrix}
    x' \\
    y' \\
    z' \\
    1
\end{bmatrix} =
\begin{bmatrix}
    1 & 0 & 0 & 0 \\
    0 & \cos \theta & -\sin \theta & 0 \\
    0 & \sin \theta & \cos \theta & 0 \\
    0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
    x \\
    y \\
    z \\
    1
\end{bmatrix}
\]

- About $y$ axis:

\[
\begin{bmatrix}
    x' \\
    y' \\
    z' \\
    1
\end{bmatrix} =
\begin{bmatrix}
    \cos \theta & 0 & \sin \theta & 0 \\
    0 & 1 & 0 & 0 \\
    -\sin \theta & 0 & \cos \theta & 0 \\
    0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
    x \\
    y \\
    z \\
    1
\end{bmatrix}
\]
Roll-Pitch-Yaw
\[ R = \text{rot}(\hat{i}, \alpha)\text{rot}(\hat{j}, \beta)\text{rot}(\hat{k}, \varphi) \]

Euler Angles
\[ R = \text{rot}(\hat{k'}', \alpha)\text{rot}(\hat{j}', \beta)\text{rot}(\hat{k}, \varphi) \]

Rotation
- Rotation by angle \( \theta \) about \((k_x, k_y, k_z)\), a unit vector
- (Rodrigues Formula)
\[
\begin{bmatrix}
x'

y'

z'

1
\end{bmatrix}
=
\begin{bmatrix}
kk(1-c)+c & kk(1-c)-ks & kk(1-c)+ks & 0 \\
kk(1-c)+ks & kk(1-c)+c & kk(1-c)-ks & 0 \\
kk(1-c)-kz & kk(1-c)-ks & kk(1-c)+c & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

where \( c = \cos \theta \) & \( s = \sin \theta \)

Homogeneous Representation of Rigid Transformations
\[
\begin{bmatrix}
\delta P \\
1
\end{bmatrix}
= \begin{bmatrix}
\delta R & \delta O \\
0 & 1
\end{bmatrix}
= \begin{bmatrix}
\delta R & \delta O \\
0 & 1
\end{bmatrix}
= \begin{bmatrix}
\delta P \\
1
\end{bmatrix}
\]

Transformation represented by 4 by 4 Matrix

Block Matrix Multiplication
Given
\[
A = \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
B = \begin{bmatrix}
B_{11} & B_{12} \\
B_{21} & B_{22}
\end{bmatrix}
\]

What is \( AB \)?
\[
AB = \begin{bmatrix}
A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\
A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22}
\end{bmatrix}
\]

Camera parameters
- Issue
  - camera may not be at the origin, looking down the z-axis
  - extrinsic parameters (Rigid Transformation)
  - one unit in camera coordinates may not be the same as one unit in world coordinates
- intrinsic parameters - focal length, principal point, aspect ratio, angle between axes, etc.

\[
\begin{bmatrix}
U \\
V \\
W
\end{bmatrix}
= \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}
= \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
Z \\
1
\end{bmatrix}
\]

Camera Calibration
Given \( n \) points \( P_1, \ldots, P_n \) with known positions and their images \( p_1, \ldots, p_n \), estimate intrinsic and extrinsic camera parameters
- See textbook for how to do it.

What about light?
Getting more light – Bigger Aperture

Limits for pinhole cameras

Pinhole Camera Images with Variable Aperture

The reason for lenses

Thin Lens

Thin Lens: Center
Thin Lens: Focus

Incoming light rays parallel to the optical axis pass through the focus, F

Thin Lens: Image of Point

All rays passing through lens and starting at P converge upon P'

Thin Lens: Image of Point

\[ \frac{1}{z'} - \frac{1}{z} = \frac{1}{f} \]

Thin Lens: Image Plane

A price: Whereas the image of P is in focus, the image of Q isn’t.

Thin Lens: Aperture

• Smaller Aperture -> Less Blur
• Pinhole -> No Blur

Field of View
Deviations from the lens model

Deviations from this ideal are \textit{aberrations}.

Two types:

1. geometrical
   - spherical aberration
   - astigmatism
   - distortion
   - coma

2. chromatic

Aberrations are reduced by combining lenses:

\begin{itemize}
  \item \textit{Compound lenses}
\end{itemize}

\section*{Spherical aberration}

Rays parallel to the axis do not converge.

Outer portions of the lens yield smaller focal lengths.

\begin{itemize}
  \item $d'$
  \item $f'$
\end{itemize}

\section*{Distortion}

Magnification/focal length different for different angles of inclination.

\begin{itemize}
  \item \textit{pincushion} (telephoto)
  \item \textit{barrel} (wide-angle)
\end{itemize}

Can be corrected! (if parameters are known)

\section*{Chromatic aberration}

Index of refraction of lens depends on wavelength of light.

Rays of different wavelengths focus in different planes.

Cannot be removed completely.

Sometimes \textit{achromatization} is achieved for more than 2 wavelengths.

\section*{Spatial Non-Uniformity}

The image is blurred and appears colored at the fringe.

\begin{itemize}
  \item camera
  \item Iris
\end{itemize}
Vignetting

Only part of the light reaches the sensor
Periphery of the image is dimmer