Image Formation and Cameras

Introduction to Computer Vision
CSE 152
Lecture 3

Announcements
• Assignment 0: due Thursday
• Office Hour: Thursday 11:00-12:00
• Read Trucco & Verri: pp. 15-40

Image Formation: Outline
• Factors in producing images
• Projection
• Perspective
• Vanishing points
• Orthographic
• Lenses
• Sensors
• Quantization/Resolution
• Illumination
• Reflectance

Earliest Surviving Photograph
• First photograph on record, “la table service” by Nicephore Niepce in 1822.
• Note: First photograph by Niepce was in 1816.

How Cameras Produce Images
• Basic process:
  – photons hit a detector
  – the detector becomes charged
  – the charge is read out as brightness
• Sensor types:
  – CCD (charge-coupled device)
    • high sensitivity
    • high power
    • cannot be individually addressed
    • blooming
  – CMOS
    • most common
    • simple to fabricate (cheap)
    • lower sensitivity, lower power
    • can be individually addressed

Images are two-dimensional patterns of brightness values.
Effect of Lighting: Monet

Change of Viewpoint: Monet

Haystack at Chaillly at Sunrise (1865)

Pinhole Camera: **Perspective projection**

- Abstract camera model - box with a small hole in it

Camera Obscura

- Used to observe eclipses (eg., Bacon, 1214-1294)
- By artists (eg., Vermeer).

Camera Obscura

- Jetty at Margate England, 1898.

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http://www.acmi.net.au/AIC/CAMERA_OBSCURA.html (Russell Naughton)

http://brightbytes.com/cosite/collection2.html (Jack and Beverly Wilgus)
Distant objects are smaller

Note “intersection of rays” ala Leonardo.

Geometric properties of projection

- Points go to points
- Lines go to lines
- Planes go to whole image or half-plane
- Polygons go to polygons
- Angles & distances not preserved

- Degenerate cases:
  - line through focal point yields point
  - plane through focal point yields line

Parallel lines meet in the image

- The projection of parallel lines meet at the vanishing point
- Intersection of line through O parallel to the 3-D line(s)
- A single line can have a vanishing point

Take out paper and pencil

Virtual Image Plane
Different 3D directions correspond to different vanishing points.

The equation of projection:

\[ (x, y, z) \rightarrow \left( \frac{fx}{z}, \frac{fy}{z}, -f \right) \]

Ignoring the third coordinate, we get:

A Digression

Homogenous Coordinates and Camera Matrices

What is the intersection of two lines in a plane?

Do two lines in the plane always intersect at a point?

No, Parallel lines don’t meet at a point.
Can the perspective image of two parallel lines meet at a point?

**YES**

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**Projective geometry** provides an elegant means for handling these different situations in a unified way and **homogenous coordinates** are a way to represent entities (points & lines) in projective spaces.

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**Homogenous coordinates**

- Our usual coordinate system is called a Euclidean or affine coordinate system
- Rotations, translations and projection in Homogenous coordinates can be expressed linearly as matrix multiplies

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**Projective Geometry**

- Axioms of Projective Plane
  1. Every two distinct points define a line
  2. Every two distinct lines define a point (intersect at a point)
  3. There exists three points, A,B,C such that C does not lie on the line defined by A and B.
- Different than Euclidean (affine) geometry
- Projective plane is “bigger” than affine plane – includes “line at infinity”

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**Homogenous coordinates**

A way to represent points in a projective space

1. Add an extra coordinate
   e.g., \((x,y) \rightarrow (x,y,1) = (u,v,w)\)
2. Impose equivalence relation such that \((\lambda \not= 0)\)
   \((u,v,w) \sim \lambda(u,v,w)\)
   i.e., \((x,y,1) = (\lambda x, \lambda y, \lambda)\)
3. “Point at infinity” – zero for last coordinate
   e.g., \((x,y,0)\)

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**Changes of coordinates:**

**Euclidean -> Homogenous** -> **Euclidean**

In 2-D
- Euclidean -> Homogenous: \((x, y) \rightarrow k (x,y,1)\)
- Homogenous -> Euclidean: \((u,v,w) \rightarrow (u/w, v/w)\)

In 3-D
- Euclidean -> Homogenous: \((x, y, z) \rightarrow k (x,y,z,1)\)
- Homogenous -> Euclidean: \((u, v, w, t) \rightarrow (u/w, v/w, z/w)\)
The camera matrix 

\[(x, y, z) \rightarrow \left( \frac{f}{z} x, \frac{f}{z} y \right)\]

Turn this expression into homogenous coordinates

- HC’s for 3D point are \((X, Y, Z, T)\)
- HC’s for point in image are \((U, V, W)\)

\[
\begin{bmatrix}
U \\
V \\
W \\
T
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & \frac{1}{f} & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Perspective Camera Matrix
A 3x4 matrix

Simplified Camera Models

Perspective Projection

Affine Camera Model

Scaled Orthographic Projection

Orthographic Projection

Affine Camera Model

Appropriate in Neighborhood About \((x_0, y_0, z_0)\)

- Take Perspective projection equation, and perform Taylor Series Expansion about some point \((x_0, y_0, z_0)\).
- Drop terms of higher order than linear.
- Resulting expression is the affine camera model

\[
\begin{bmatrix}
\frac{1}{x_0} & 0 & -x_0/z_0 & 1/3 \\
0 & \frac{1}{y_0} & -y_0/z_0 & 1/3 \\
0 & 0 & \frac{1}{z_0} & 1/3 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix} = Ap + b
\]

Rewrite Affine camera model in terms of Homogenous Coordinates
Consider doing an expansion about a point $X_0 = 0, Y_0 = 0$

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1/z_0 & 0 & -x_0/z_0^2 & x_0/z_0 \\ 0 & 1/z_0 & -y_0/z_0^2 & y_0/z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} \approx \begin{bmatrix} 1/z_0 & 0 & 0 & 0 \\ 0 & 1/z_0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

The projection matrix for orthographic projection

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} 1/z_0 & 0 & 0 & 0 \\ 0 & 1/z_0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Unlike perspective,
- Parallel lines project to parallel lines
- Ratios of distances are preserved under orthographic

Orthographic projection

Starting with Affine camera mode
Take Taylor series about $(0, 0, z_0)$ – a point on optical axis

Other camera models
- Generalized camera – maps points lying on rays and maps them to points on the image plane.

Some Alternative “Cameras”