Photometric Stereo

Announcements

- HW3 due Tuesday
- HW4 to be assigned on Tuesday
- Changing order of topics: Now
  - Photometric stereo
  - Recognition
  - Motion

Shading reveals 3-D surface geometry

Multi-view stereo vs. Photometric Stereo: Assumptions

- Multi-view (binocular) Stereo
  - Multiple images
  - Static scene
  - Multiple viewpoints
  - Fixed lighting
- Photometric Stereo
  - Multiple images
  - Static scene
  - Fixed viewpoint
  - Multiple lighting conditions

- Shape-from-shading: Use just one image to recover shape. Requires knowledge of light source direction and BRDF everywhere. Too restrictive to be useful.

- Photometric stereo: Single viewpoint, multiple images under different lighting.
  1. Arbitrary known BRDF
  2. Lambertian BRDF, known lighting
  3. Lambertian BRDF, unknown lighting.

An example of photometric stereo
BRDF

- Bi-directional Reflectance Distribution Function
  \[ \rho(\theta_{\text{in}}, \phi_{\text{in}} ; \theta_{\text{out}}, \phi_{\text{out}}) \]
- Function of
  - Incoming light direction: \( \theta_{\text{in}}, \phi_{\text{in}} \)
  - Outgoing light direction: \( \theta_{\text{out}}, \phi_{\text{out}} \)
- Ratio of incident irradiance to emitted radiance

Photometric Stereo:
General BRDF and Reflectance Map

Coordinate system

Gradient Space (p,q)

Image Formation

For a given point A on the surface, the image irradiance \( E(x,y) \) is a function of
1. The BRDF at A
2. The surface normal at A
3. The direction of the light source
Let the BRDF be the same at all points on the surface, and let the light direction be constant.

1. Then image irradiance is a function of only the direction of the surface normal.
2. In gradient space, we can write $E(p,q)$.
3. We can measure $E(p,q)$ by taking an image of a sphere made of a single material under distant lighting.

Example Reflectance Map:
- Lambertian surface
- For lighting from front

Reflectance Map of Lambertian + Specular Surface

Reflectance Map of Lambertian Surface

What does the intensity (irradiance) of one pixel in one image tell us?
- It constrains the surface normal projecting to that point to a curve.

Two Light Sources
- Two reflectance maps
- A third image would disambiguate match
Photometric stereo:
Step 1
Offline:
Use known source direction, geometry & BRDF to construct reflectance map.
Online:
1. Acquire three images with known light source direction.
2. For each pixel location \((x, y)\), find \((p, q)\) as the intersection of the three curves.
3. This is the surface normal at pixel \((x, y)\). Over image, this is normal field.

Plastic Baby Doll: Normal Field

Next step:
Go from normal field to surface

Recovering the surface \(f(x, y)\)

Many methods: Simplest approach
1. From normal field \(n = (n_x, n_y, n_z)\), \(p = -n_x/n_z\), \(q = -n_y/n_z\).
2. Integrate \(p = df/dx\) along a row \((x, 0)\) to get \(f(x, 0)\).
3. Then integrate \(q = df/dy\) along each column starting with value of the first row.

What might go wrong?
• Height \(z(x, y)\) is obtained by integration along a curve from \((x_0, y_0)\).
  \[ z(x, y) = z(x_0, y_0) + \int_{(x_0)}^{(x, y)} (pdx + qdy) \]
• If one integrates the derivative field along any closed curve, one expects to get back to the starting value.
• Might not happen because of noisy estimates of \((p, q)\).
What might go wrong?

Integrability. If \( z(x,y) \) is the height function, we expect that

\[
\frac{\partial z}{\partial y} - \frac{\partial z}{\partial x} = \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}.
\]

In terms of estimated gradient space \((p,q)\), this means:

\[
\frac{\partial p}{\partial y} = \frac{\partial q}{\partial x}.
\]

But since \( p \) and \( q \) were estimated independently at each point as intersection of curves on three reflectance maps, equality is not going to exactly hold.

II. Photometric Stereo: Lambertian Surface, Known Lighting

Lambertian Surface

At image location \((u,v)\), the intensity of a pixel \( x(u,v) \) is:

\[
e(u,v) = [a(u,v) \cdot n(u,v)] \cdot s_0 = b(u,v) \cdot s.
\]

where

- \( a(u,v) \) is the albedo of the surface projecting to \((u,v)\).
- \( n(u,v) \) is the direction of the surface normal.
- \( s_0 \) is the light source intensity.
- \( s \) is the direction to the light source.

If the light sources \( s_1, s_2, \) and \( s_3 \) are known, then we can recover \( b \) from as few as three images. (Photometric Stereo: Silver 80, Woodham 81).

\[
[e_1 \ e_2 \ e_3] = b^T [s_1 \ s_2 \ s_3]
\]

- i.e., we measure \( e_1, e_2, \) and \( e_3 \) and we know \( s_1, s_2, \) and \( s_3 \). We can then solve for \( b \) by solving a linear system.

\[
b^T = [e_1 \ e_2 \ e_3] [s_1 \ s_2 \ s_3]^T
\]

- Normal is: \( n = b/|b| \), albedo is: \(|b|\)

What if we have more than 3 Images?

Linear Least Squares

Let the residual be \( r = e - Sb \)

Rewrite as

\[
e = Sb
\]

where

- \( e \) is \( n \) by \( 1 \)
- \( b \) is \( 3 \) by \( 1 \)
- \( S \) is \( n \) by \( 3 \)

Solving for \( b \) gives

\[
b = (S^T S)^{-1} S^T e
\]

Input Images
Recovered albedo

Recovered normal field

Surface recovered by integration

Lambertian Photometric Stereo

Reconstruction with albedo map

Without the albedo map
Another person

No Albedo map

III. Photometric Stereo with unknown lighting and Lambertian surfaces

How do you construct subspace?

[\begin{bmatrix} e_1 & e_2 & e_3 \end{bmatrix} = B^T \begin{bmatrix} s_1 & s_2 & s_3 \end{bmatrix}]

• Given three or more images $e_1, e_2, \ldots$, estimate $B$ and $s_i$.
• How? Given images in form of $E = [e_1, e_2, \ldots]$, Compute SVD(E) and $B^*$ is $n$ by $3$ matrix formed by first $3$ singular values.

Matrix Decompositions

• Definition: The factorization of a matrix $M$ into two or more matrices $M_1, M_2, \ldots, M_n$, such that $M = M_1 M_2 \ldots M_n$.
• Many decompositions exist…
  – QR Decomposition
  – LU Decomposition
  – LDU Decomposition
  – Etc.

Singular Value Decomposition

Excellent ref: “Matrix Computations,” Golub, Van Loan
• Any $m$ by $n$ matrix $A$ may be factored such that
  $$ A = U \Sigma V^T $$
  $$ [m \times n] = [m \times m][m \times n][n \times n] $$

  • $U$: $m$ by $m$, orthogonal matrix
    – Columns of $U$ are the eigenvectors of $AA^T$
  • $V$: $n$ by $n$, orthogonal matrix,
    – columns are the eigenvectors of $A^TA$
  • $\Sigma$: $m$ by $n$, diagonal with non-negative entries $(\sigma_1, \sigma_2, \ldots, \sigma_s)$ with $s = \min(m,n)$ are called the called the singular values
    – Singular values are the square roots of eigenvalues of both $AA^T$ and $A^TA$
  – Result of SVD algorithm: $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_s$
Applying SVD to Photometric stereo

- The images are formed by

\[
\begin{bmatrix}
  e_1, e_2, \ldots, e_n
\end{bmatrix} = B^T \begin{bmatrix}
  s_1, s_2, \ldots, s_n
\end{bmatrix}
\]

- So, \( \text{svd}(E) = \begin{bmatrix}
  U & \Sigma & V^T
\end{bmatrix} \) where \( U \) is \( N \) by \( n \), \( \Sigma \) is \( n \) by \( n \), and \( V^T \) is \( n \) by \( N \).
- Without noise, we expect 3 non-zero singular values, and so \( U \Sigma V^T = U \Sigma' V'^T \) where \( U' \) is \( N \) by 3, \( \Sigma' \) is 3 by 3, and \( V'^T \) is 3 by \( n \).
- In particular \( B = U'A \) where \( A \) is some 3x3 matrix.

Do Ambiguities Exist? Yes

- Is \( B \) unique?
- For any \( A \in \text{GL}(3) \), \( B^* = BA \) also a solution
- For any image of \( B \) produced with light source \( S \), the same image can be produced by lighting \( B^* \) with \( S^* = A^{-1} S \) because

\[
X = B^*S^* = BA A^{-1}S = BS
\]
- When we estimate \( B \) using SVD, the rows are NOT generally normal * albedo.

Surface Integrability

In general, \( B^* \) does not have a corresponding surface.

Linear transformations of the surface normals in general do not produce an integrable normal field.

Only Generalized Bas-Relief transformations satisfy the integrability constraint:

\[
A = \begin{bmatrix}
  -1 & 0 & \lambda \\
  -\mu & 1 & 0 \\
  -\nu & 0 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
  f(x,y) + \mu u + \nu v
\end{bmatrix}
\]

Generalized Bas-Relief Transformations

Objects differing by a GBR have the same illumination cone.

Without knowledge of light source location, one can only recover surfaces up to GBR transformations.

Uncalibrated photometric stereo

1. Take \( n \) images as input, perform SVD to compute \( B^* \).
2. Find some \( A \) such that \( B^*A \) is close to integrable.
3. Integrate resulting gradient field to obtain height function \( f^*(x,y) \).

Comments:
- \( f^*(x,y) \) differs from \( f(x,y) \) by a GBR.
- Can use specularities to resolve GBR for non-Lambertian surface.