Stereo Vision III

Introduction to Computer Vision
CSE 152
Lecture 15

Announcements

• HW3 due Tuesday, May 25

Stereo Vision Outline

• Offline: Calibrate cameras & determine “epipolar geometry”

• Online
  1. Acquire stereo images
  2. Rectify images to convenient epipolar geometry
  3. Establish correspondence
  4. Estimate depth

Epipolar Constraint: Calibrated Case

\[
\bar{O}p, [\bar{O}\bar{O} \times \bar{O}p] = 0 \quad \Rightarrow \quad p \cdot [\bar{O} \times [R \bar{p}]] = 0
\]

\[
p = \begin{pmatrix} u, v, 1 \end{pmatrix}^T
\]

\[
p' = \begin{pmatrix} u', v', 1 \end{pmatrix}^T
\]

\[M = \begin{pmatrix} \bar{R} & \bar{O}^T \end{pmatrix}
\]

\[
M' = \begin{pmatrix} R^T & \bar{O}^T \end{pmatrix}
\]

Epipolar Constraint (Longuet-Higgins, 1981)

\[
p^T \bar{E} p' = 0 \quad \text{with} \quad \bar{E} = [\bar{I}, \bar{R}]
\]

The Eight-Point Algorithm (Longuet-Higgins, 1981)

\[
\begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix} = 0
\]

\[
\begin{bmatrix} u_1, u_2, u_3, v_1, v_2, v_3, x_1, x_2, x_3, 1 \end{bmatrix} = 0
\]

Set \( F_{33} \) to 1

Minimize:

\[
\sum \left( p^T F p' \right)^2
\]

under the constraint \( \|F\|^2 = 1 \)
Properties of the Essential Matrix

- \( E \) is the epipolar line associated with \( p' \).
- \( E'p \) is the epipolar line associated with \( p \).
- \( Ee=0 \) and \( E'e=0 \).
- The epipole is the Eigenvector of \( E \) corresponding to the zero eigenvalue.
- \( E \) is singular.
- \( E \) has two equal non-zero singular values (Huang and Faugeras, 1989).

Fundamental vs. Essential Matrix

- Both are 3x3 matrices used in the epipolar constraint (a bilinear equation).
- Essential Matrix: Calibrated Cameras, Can be generated as \([t;]R\) or factored into \([t;]\) and \(R\).
- Fundamental Matrix based on pixel coordinates in homogenous coordinates, not rays in 3-D. Doesn’t factor into \(R\) & \([t;]\).
- Singular matrices.
- Can be used to find epipoles and corresponding epipolar lines.

Rectification

Given a pair of images, transform both images so that epipolar lines are scan lines.

Image pair rectification

- Simplify stereo matching by warping the images.
- Apply projective transformation so that epipolar lines correspond to horizontal scanlines.
- \( \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - He \) map epipole \( e \) to (1,0,0).
- Try to minimize image distortion.
- Note that rectified images usually not rectangular.
Rectification
Given a pair of images, transform both images so that epipolar lines are scan lines.

Rectified Images
See Section 7.3.7 for specific method

Features on same epipolar line

Mobi: Stereo-based navigation

Epipolar correspondence
This version is feature-based: detect edges in 1-D signal, and use dynamic programming to find correspondences that minimize an error function.

Symbolic Map

A challenge: Multiple Interpretations
Each feature on left epipolar line match one and only one feature on right epipolar line.
Multiple Interpretations

Each feature on left epipolar line match one and only one feature on right epipolar line.

Dense Correspondence:
A Photometric constraint
• Same world point has same intensity in both images (Constant Brightness Constraint)
  – Lambertian fronto-parallel
  – Issues:
    • Noise
    • Specularity
    • Foreshortening

Using epipolar & constant Brightness constraints for stereo matching

For each epipolar line
  For each pixel in the left image
  • compare with every pixel on same epipolar line in right image
  • pick pixel with minimum match cost
  • This will never work, so:

Improvement: match windows
(Seitz)
Slide Window to different disparities to find best match

Comparing Windows:

\[
SSD = \sum_{[i,j] \in R} (f(i,j) - g(i,j))^2
\]

\[
C_{fg} = \sum_{[i,j] \in R} f(i,j)g(i,j)
\]

For each window, match to closest window on epipolar line in other image.

(Camps)

Best Match amounts to minimizing (or maximizing some) Match Metric

Correspondence Search Algorithm

For \( i = 1:nrows \) for \( j=1:ncols \)
\[
\text{best}(i,j) = -1
\]
for \( k = \text{mindisparity}:\text{maxdisparity} \)
\[
c = \text{Match\_Metric}(l_1(i,j),l_2(i,j+k),\text{winsize})
\]
if \((c > \text{best}(i,j))\)
\[
\text{best}(i,j) = c
\]
\[
\text{disparities}(i,j) = k
\]
end
end
O(nrows * ncols * disparities * winx * winy)

Match Metric Summary

<table>
<thead>
<tr>
<th>MATCH METRIC</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>NCC</td>
<td>Normalized Cross-Correlation (AXE)</td>
</tr>
<tr>
<td>SSD</td>
<td>Sum of Squared Differences (SSD)</td>
</tr>
<tr>
<td>NSSD</td>
<td>Normalized SSD</td>
</tr>
<tr>
<td>SAD</td>
<td>Sum of Absolute Differences (SAD)</td>
</tr>
<tr>
<td>Zero Mean SAD</td>
<td></td>
</tr>
<tr>
<td>Census</td>
<td>These two are actually the same</td>
</tr>
</tbody>
</table>

Stereo results

– Data from University of Tsukuba

Results with window correlation

Window-based matching (best window size) (Seitz)
Results with better method

State of the art method

Ground truth

Boykov et al.,
Fast Approximate Energy Minimization via Graph Cuts,
International Conference on Computer Vision, September 1999.

(Stateiz)

Some Issues

• Ambiguity
• Window size
• Window shape
• Lighting
• Half occluded regions

Window size

W = 3
W = 20

Better results with adaptive window

T. Kanade and M. Okutomi,
A Stereo Matching Algorithm with an Adaptive Window: Theory and Experiment,

D. Scharstein and R. Szeliski.
Stereo matching with nonlinear diffusion.

Effect of window size (Seitz)

Window Shape and Forshortening

Window Shape: Fronto-parallel Configuration
Lighting Conditions (Photometric Variations)

Problem of Occlusion

Stereo Constraints

Stereo Matching using Dynamic Programming

Stereo matching
Some Challenges & Problems

• Photometric issues:
  – specularities
  – strongly non-Lambertian BRDF’s

• Surface structure
  – lack of texture
  – repeating texture within horopter bracket

• Geometric ambiguities
  – as surfaces turn away, difficult to get accurate reconstruction (affine approximate can help)
  – at the occluding contour, likelihood of good match but incorrect reconstruction

Variations on Binocular Stereo

1. Trinocular Stereopsis
2. Helmholtz Reciprocity Stereopsis

Trinocular Epipolar Constraints

\[
\begin{align*}
    p_1^{e_1}c_1c_2 &= 0 \\
    p_2^{e_2}c_2c_3 &= 0 \\
    p_3^{e_3}c_3c_1 &= 0 \\
    e_1^{e_1}e_2^{e_2} - e_2^{e_2}e_3^{e_3} - e_3^{e_3}e_1^{e_1} &= 0
\end{align*}
\]

These constraints are not independent!

More on stereo …

The Middleburry Stereo Vision Research Page
http://cat.middlebury.edu/stereo/

Recommended reading
