Analyzing loops-10pts Consider the following algorithm, which takes as input two sorted (from smallest to largest) arrays A and B of size n and for each 1 ≤ I ≤ n computes the minimum 1 ≤ J ≤ n for which B[J] > A[I] and stores it in C[I]. (If no such J exists, it sets C[I] equal to n + 1.) Give a time analysis of the algorithm, up to Θ.

SmallestLargerElement(A[1..n], B[1..n])
1. J ← 1
2. FOR I=1 to n do:

Greedy Algorithms and use of data structures in algorithms : 30 points total
Consider the following problem: You are making probes of an asteroid to reveal its chemical composition, by shooting lasers at certain frequencies at the asteroid. Each substance s_i on a list of n possible constituent substances has a range [l_i, h_i] where it will react to the laser. You need to pick the smallest set of frequencies f_1, . . . f_k so that each s_i will react to one of the frequencies f_j, i.e., for each i there is at least one j with l_i ≤ f_j ≤ h_i.

Part 1 : 5 points for counter-example Below is a greedy strategy for this problem that is not guaranteed to produce optimal solutions. Show that it fails to produce the optimal solution on the following input: [0,6], [1,6], [2, 6], [3, 5], [6,11],[6,10],[8,9].
Candidate Strategy one: Pick a frequency f that the maximum number of substances react to. Remove substances that react to f and recurse, until all substances are covered.

Part 2: 5 pts Here is a greedy strategy that does produce optimal solutions. Illustrate the algorithm on the counter-example from Part 1.
Candidate Strategy two: Let S_i be the substance with the smallest value of h_i. Pick frequency f = h_i. Remove substances that react to f, and recurse, until all substances are covered.
Part 2: 10 points For the optimal strategy, Candidate Strategy 2, prove that it is correct. You can use any valid proof method, but here are two possible outlines as hints:

Approach 1: Achieves the bound: Let $S_1, S_2, \ldots S_k$ be the substances chosen by the greedy strategy (i.e., $S_1$ has the smallest $h_i$, $S_2$ has the smallest $h_i$ once substances reacting to $h_1$ are removed, etc.) First prove that the corresponding intervals $[l_i, h_i]$ are disjoint. Use this to prove that any solution requires at least $k$ frequencies.

Approach 2: Greedy stays ahead: Let $\{f_1, \ldots, f_t\}$ be the greedy set of frequencies, in increasing order, and let $\{f'_1, f'_t\}$ be any other set of frequencies, also in increasing order. Prove by induction that $f_i \geq f'_i$. Then use this to show that $t \leq t'$.

Part 4: 10 points For the optimal strategy (Strategy 2), describe an efficient algorithm that carries out the strategy. Your description should specify which data structures you use, and any pre-processing steps. Give a time analysis.