

CSE105 (spring 2008): Homework 1

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Solutions

1 Review

- Let A and B be the sets $A = \{x, y, z\}$ and $B = \{x, y\}$.
 - No, A is not a subset of B .
 - Yes, B is a subset of A .
 - $A \cup B = A = \{x, y, z\}$.
 - $A \cap B = B = \{x, y\}$.
 - $A \times B = \{(x, x), (x, y), (y, x), (y, y), (z, x), (z, y)\}$.
 - $\mathcal{P}(A) = \{\emptyset, \{x\}, \{y\}, \{z\}, \{x, y\}, \{x, z\}, \{y, z\}, \{x, y, z\}\}$.
 - No, $\mathcal{P}(A \times B) \neq \mathcal{P}(A) \times \mathcal{P}(B)$.
- If X is a finite set with n elements, then $\mathcal{P}(X)$ has 2^n elements.

2 Formal Description of a DFA

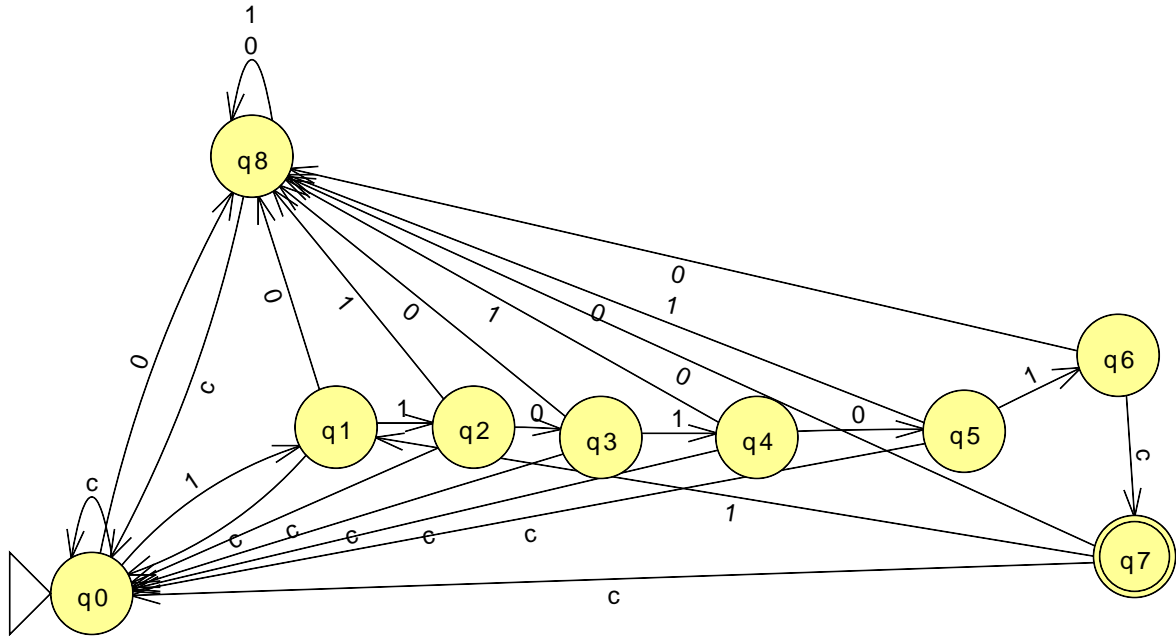
The strings ε , **uuddud** and **duddud** are accepted by M and **dduddu** is rejected. The DFA M counts up (towards 1) or down (towards 5) on a **d** or **u**, respectively, not going outside the range 1–5. It accepts if the count ends at 3 or 5.

3 Simple DFAs

- The start state of M_1 is q_0 .
- The set of accept states of M_1 is $\{q_1\}$.
- The start state of M_2 is q_0 .
- The set of accept states of M_2 is $\{q_0, q_3\}$.
- On input **aabb**, M_1 goes through q_0, q_1, q_2, q_0, q_0 .
- The input **aabb** is not accepted by M_1 .
- On input **baaba**, M_2 goes through $q_0, q_1, q_2, q_1, q_3, q_2$.
- The input **baaba** is not accepted by M_2 .
- The empty string is not accepted by M_1 .
- The empty string is accepted by M_2 .

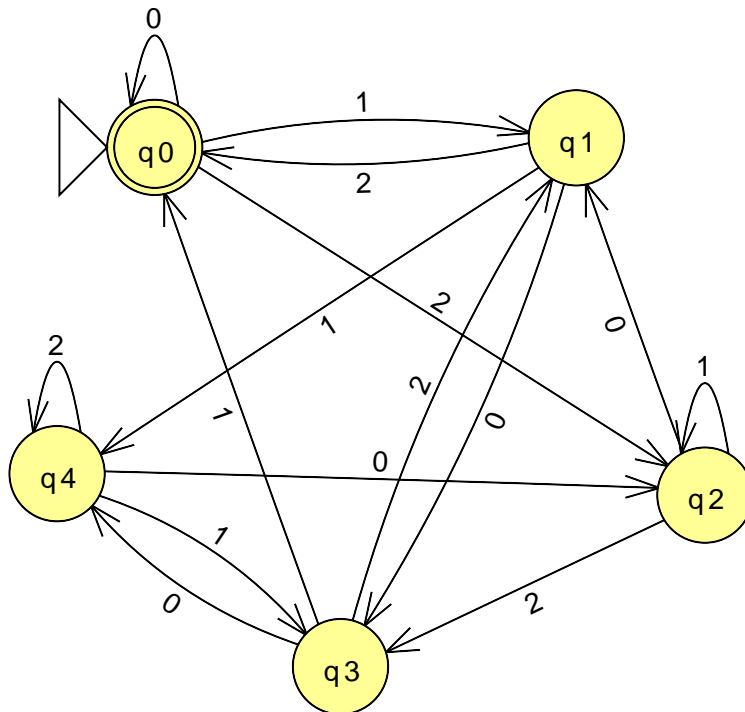
4 Keycode Checker

We need 7 states to keep track of what input symbol is expected next, a state to handle incorrect inputs and an accept state.



5 Ternary Numbers mod 5

Each of the five states represents the number as seen so far, modulo 5. Each time a new trit¹ is encountered, we multiply the current value by 3 and add the value of the trit.



¹A trit is a ternary digit.