1 Review

1. Let \( A \) and \( B \) be the sets \( A = \{x, y, z\} \) and \( B = \{x, y\} \).
   
   (a) No, \( A \) is not a subset of \( B \).
   
   (b) Yes, \( B \) is a subset of \( A \).
   
   (c) \( A \cup B = A = \{x, y, z\} \).
   
   (d) \( A \cap B = B = \{x, y\} \).
   
   (e) \( A \times B = \{(x, x), (x, y), (y, x), (y, y), (z, x), (z, y)\} \).
   
   (f) \( P(A) = \{\emptyset, \{x\}, \{y\}, \{z\}, \{x, y\}, \{x, z\}, \{y, z\}, \{x, y, z\}\} \).
   
   (g) No, \( P(A \times B) \neq P(A) \times P(B) \).

2. If \( X \) is a finite set with \( n \) elements, then \( P(X) \) has \( 2^n \) elements.

2 Formal Description of a DFA

The strings \( \varepsilon, uuddud \) and \( duddud \) are accepted by \( M \) and \( dduddu \) is rejected. The DFA \( M \) counts up (towards 1) or down (towards 5) on a \( d \) or \( u \), respectively, not going outside the range 1–5. It accepts if the count ends at 3 or 5.

3 Simple DFAs

1. The start state of \( M_1 \) is \( q_0 \).
2. The set of accept states of \( M_1 \) is \( \{q_1\} \).
3. The start state of \( M_2 \) is \( q_0 \).
4. The set of accept states of \( M_2 \) is \( \{q_0, q_3\} \).
5. On input \( aabb \), \( M_1 \) goes through \( q_0, q_1, q_2, q_0, q_0 \).
6. The input \( aabb \) is not accepted by \( M_1 \).
7. On input \( baaba \), \( M_2 \) goes through \( q_0, q_1, q_2, q_1, q_3, q_2 \).
8. The input \( baaba \) is not accepted by \( M_2 \).
9. The empty string is not accepted by \( M_1 \).
10. The empty string is accepted by \( M_2 \).

4 Keycode Checker

We need 7 states to keep track of what input symbol is expected next, a state to handle incorrect inputs and an accept state.
5 Ternary Numbers mod 5

Each of the five states represents the number as seen so far, modulo 5. Each time a new trit\(^1\) is encountered, we multiply the current value by 3 and add the value of the trit.

\(^1\)A trit is a ternary digit.