Midterm Exam

Name: ___________________________
ID: ___________________________

Be clear and concise. Write your answers in the space provided. Use the backs of pages for scratchwork.
1. **(10 points)** Consider the following graph.

(a) Do a depth-first search of the above graph, processing nodes (and edges out of a node) in alphabetical order. Show the DFS search tree/forest, and write in the pre and post numbers.

(b) Draw the DAG (metagraph) of strongly connected components.

(c) What is the minimum number of edges you have to add to make the graph strongly connected?
2. (10 points) Give an efficient algorithm which takes as input a directed graph \( G = (V, E) \), and determines whether or not there is a vertex \( s \in V \) from which all other vertices are reachable. [Hint: your algorithm should take linear time.]

**Algorithm:**

Briefly justify its correctness:

Briefly justify its running time:
3. (10 points) Often there are multiple shortest paths between two nodes of a graph. Give a linear-time algorithm for the following task.

*Input:* Undirected graph \( G = (V, E) \) with unit edge lengths; nodes \( u, v \in V \).

*Output:* The number of distinct shortest paths from \( u \) to \( v \).

**Algorithm:**

Briefly justify its correctness:

Briefly justify its running time:
4. (10 points) The following statements are false. Give a counterexample in each case.

(a) When depth-first search is run on a directed acyclic graph, the node with highest PRE number is always a sink.

(b) If $G$ is a connected, undirected graph, then its BFS search tree contains the shortest path between every pair of nodes.

(c) Any directed acyclic graph with $n$ nodes can have at most $n$ distinct topological orderings.

(d) If $f(n)$ and $g(n)$ are functions for which $f = O(g)$, then $2^f = O(2^g)$.

(e) If a graph is not strongly connected, then its depth-first search will necessarily have a cross edge.