Every arc has a length

The bottom path is better
Maximum Flow Problem

Every arc has a length

The max flow in the bottom path is less than the top path
Concept of bottleneck in a path is generalized to a network

Max flow is \((2 + 1) = 3\)
Max flow is \((3 + 3) = 6\)
Maximum Flow Problem

Given a network

with a source $V_S$, a sink $V_T$

arcs with capacity $b_{ij}$

What is the Max Flow from $V_S$ to $V_T$

if flow is conserved in

all nodes except $V_S \& V_T$
Let $x_{ij}$ be the amount of arc flow in

\[ x_{ij} \leq b_{ij} \]
Constraints

The total flow into a node \( j \) must be equal to the total flow out of a node

\[
\sum_i x_{ij} - \sum_k x_{jk} = 0
\]
Example

capacity $b_{ij}$

Arc flow $x_{ij}$
Max  \[ V \]

Sub
\[
\sum_{i} x_{ij} - \sum_{k} x_{jk} = \begin{cases} 
-v & j = s \\
0 & j \neq s, t \\
v & j = t 
\end{cases}
\]

\[ 0 \leq x_{ij} \leq b_{ij} \]
The MAX FLOW value in a network is equal to the value of a min cut separating the source and the sink.

\[ \text{MAX FLOW} = \text{MIN CUT} \]
Max flow Min cut

MAX FLOW VALUE is unique, but flow pattern is not.

There could be many min cuts in a network.
Max flow Min cut

\[ F(s, t) = c(X, \overline{X}) \]

\[ s \in X, t \in \overline{X} \]