Give proofs for each problem. Proofs can be high-level, but be precise. You may use without giving a proof any result proved in class or in the textbook.

Indirect arguments: It is an open problem whether $P \subseteq \text{DSPACE}(n)$ and it is also open whether $\text{DSPACE}(n) \subseteq P$. Nevertheless, prove that $P \neq \text{DSPACE}(n)$. (Hint: show that $\text{DSPACE}(n)$ is not closed under polynomial-time reducibility using the Space Hierarchy Theorem, Theorem 9.3, page 337).

2-SAT: The 2-SAT problem is to decide Satisfiability for CNF formulas with at most 2 literals per clause. Prove that 2-\textit{SAT} is co-\textit{NL}-complete (under deterministic logspace many-one reductions ). You can use without proof that \textit{PATH} is \textit{NL}-complete.

Counting classes Consider the problem of counting the number of accepting runs of a non-deterministic Turing machine $N$ on input $x$. Show that if $N$ uses $O(\log n)$ space, then the associated counting problem is in \textit{FP}.

Probabilistic complexity: Remember that for complexity class $C$, $P^C$ is the class of all decision problems poly-time Turing reducible to $L$ for some $L \in C$.

a. Prove that $P^{BPP} = BPP$.

b. Prove that $P^{ZPP} = ZPP$.

c. Prove that $RP = ZPP$ if and only if $P^{RP} = RP$.

Cryptography: A function $f$ is a one-way permutation if

1. $|f(x)| = |x|$
2. $f$ is 1-1, $f(x) = f(y) \rightarrow x = y$.
3. $f \in P$
4. For every polynomial-time algorithm $A$, and every polynomial $p$, the probability that, for $x \in \{0, 1\}^n$ chosen at random, $A(f(x)) = x$, is at most $1/p(n)$ for all sufficiently large $n$.

Prove that, if $P = NP \cap Co - NP$, then there are no one-way permutations.